

Module - I

Elemental and compound semiconductors:

Introduction:

Basically Solids are classified in to -

1. Conductors
2. Semiconductors
3. Insulators

This classification is based on conductivity

Property.

Conductors: Conductivity between $10^4 - 10^6 \Omega^{-1} \text{cm}^{-1}$.

Semiconductors: Materials having electrical conductivity between conductors ($10^4 - 10^6 \Omega^{-1} \text{cm}^{-1}$) and insulators ($10^{-18} - 10^{-8} \Omega^{-1} \text{cm}^{-1}$).

They have conductivity between $10^8 - 10^3 \Omega^{-1} \text{cm}^{-1}$.

Conductivity of a semiconductor can be varied by the process of doping. Conductivity can also be varied by changes in temperature, optical excitation and impurity content.

Elemental and compound semiconductors

Elemental SC

→ Composed of single species of atoms.

→ Column IV eg:- Si, Ge

Compound SC

→ Composed of two or more species of atoms

1.) Binary SC

It is a compound s.c with two elements.

→ Column III and V combination.

→ Combination of different column IV elements
 → Combination of column II and VI elements

List of Binary SCs

<u>IV</u> group	<u>III</u> - <u>V</u> compounds	<u>II</u> - <u>VI</u> Compounds
SiC	AlP	ZnS
	AlAs	ZnSe
SiGe	AlSb	ZnTe
	GaP	CdS
	GaAs	CdSe
	GaSb	CdTe
	InP	
	InAs	
	InSb	

Ternary SC

→ Compound SC with 3 elements

eg:- GaAsP, AlGaAs

Quaternary SC

→ Compound SC with 4 elements

eg:- AlGaP, InGaAsP

Applications:

Device	Material
Diode, Transistors	Si, Ge
FETs, Monolithic ICs	Si
LEDs	GaN, GaAs, GaP
Photo detector	CdSe, InSb
Solar cells	Si, GaAs
LASER	GaAs, AlGaAs
Flourescent Screen	ZnS

Types of Semiconductors:

1. Intrinsic s.c

Perfect s.c with no impurities or lattice defects.

- At 0K, valence band is completely filled and conduction band is empty. So there are no charge carriers at 0K.
- At higher temperature, electron-hole pairs are generated and valence band electrons are excited to conduction band, across the bandgap.
- These Electron-Hole Pairs (EHPs) are the only charge carriers in intrinsic s.c.
- The energy required to break the bond is the E_g (Bandgap energy).

n_0 = Concentration of electrons at thermal equilibrium

p_0 = Concentration of holes " " "

n_i : Intrinsic carrier concentration

$$n_0 = p_0 = n_i$$

2. Extrinsic s.c

In a s.c, charge carriers may be purposely introduced by adding impurities in it. This process is called doping.

$$n_0 \neq p_0 \neq n_i$$

a) N-type s.c

Majority carriers - Electrons

Minority carriers - holes

- Atoms with 5 valence es (column V) is added to the Si atom.

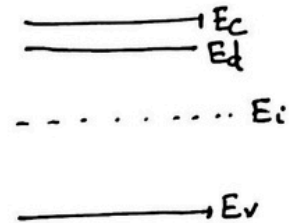
eg:- P, As, Sb

→ These N-type dopants are called Donors.

→ Donors are neutral at 0K, when occupied by $\bar{e}$ s and become +vely charged when they donate their excess valence $\bar{e}$ s to the CB.

→ So at 50-100 K, all of the $\bar{e}$ s in the impurity level are donated to CB. Such impurity level is called donor level.

$$n_0 \gg (n_i, p_0)$$



b) p type sc

Majority carriers $\rightarrow$ holes

Minority carriers $\rightarrow$ electrons

→ Atoms with 3 valence $\bar{e}$ s (column III) is added to the Si material.

eg:- B, Ga, Al act as p type dopants

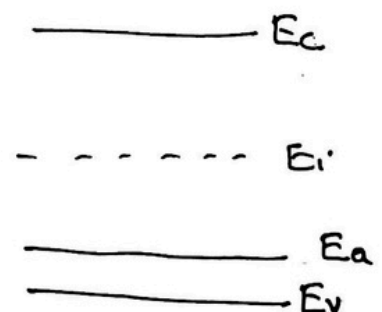
→ These p type dopants are called Acceptors.

→ In such material, there is one bond incomplete.

→ Thus at 0K, the acceptors are neutral, but at room T, they accept $\bar{e}$ s and becomes -vely charged.

→ Thus at 50-100 K, all of the $\bar{e}$ s in the impurity VB ~~level~~ are accepted by the impurity level. This impurity level is called acceptor level.

$$p_0 \gg (n_i, n_0)$$



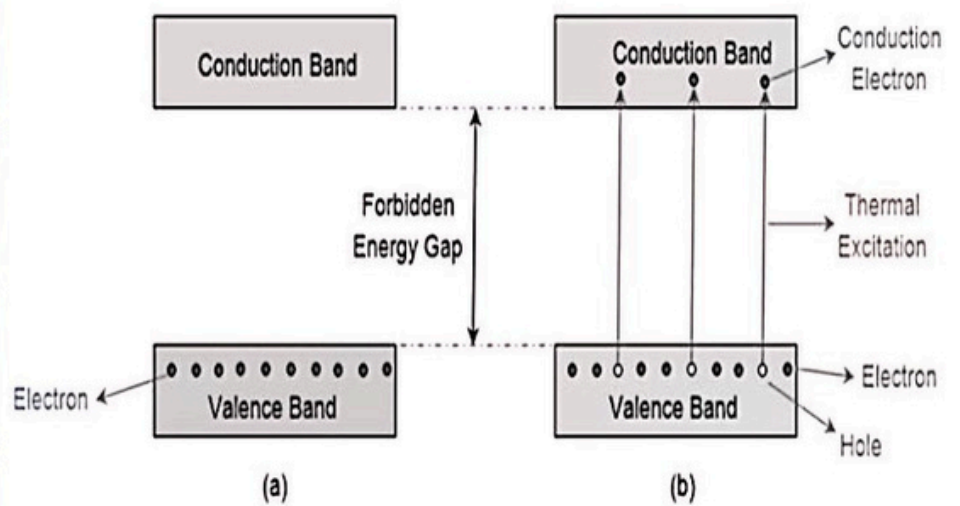
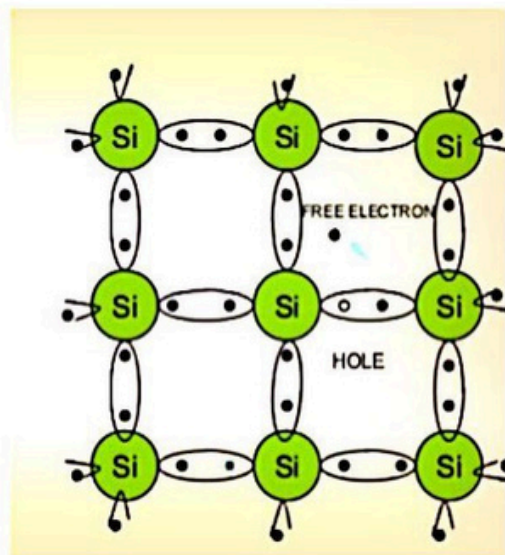
At any given temperature, if steady state carrier concentration is maintained, there is a recombination of electrons and holes (EHPs) at the same rate at which they are generated.

$$\text{Recombination rate } (r_i) = \text{Generation rate } (g_i)$$

At any temperature, the rate of recombination of electrons and holes (r_i) is proportional to the equilibrium concentration of holes (p_0) and electrons (n_0).

$$r_i = \alpha_r n_0 p_0 = \alpha_r \cdot n_i^2 = g_i$$

where α_r is the constant of proportionality depending on the recombination mechanism.



Energy Band Diagram of Intrinsic Semiconductor at (a) 0K (b) Temperature > 0K

N - Type Semiconductors:

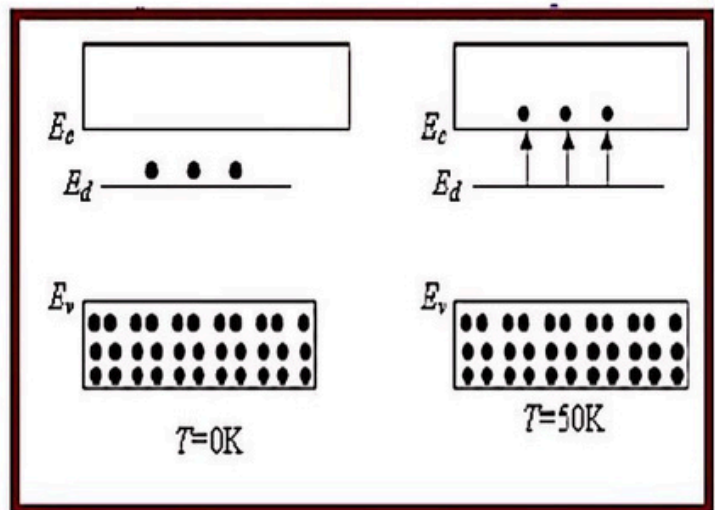
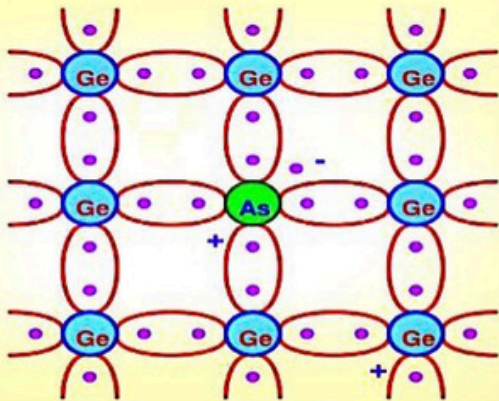
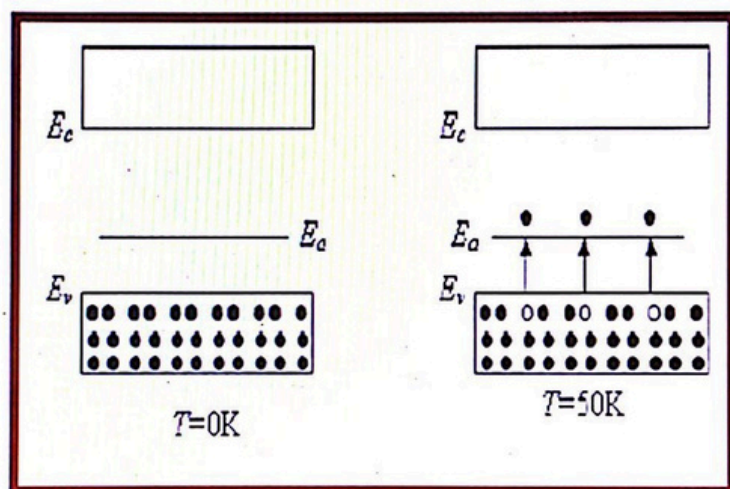
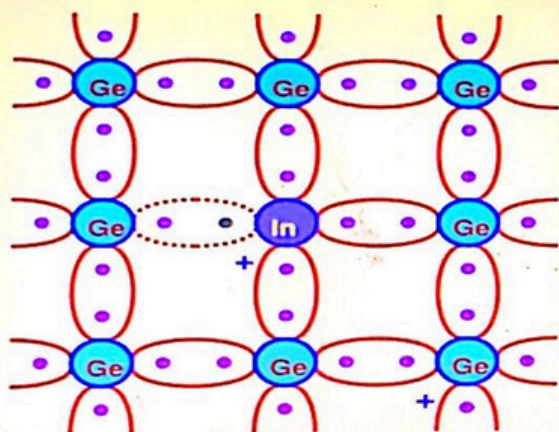


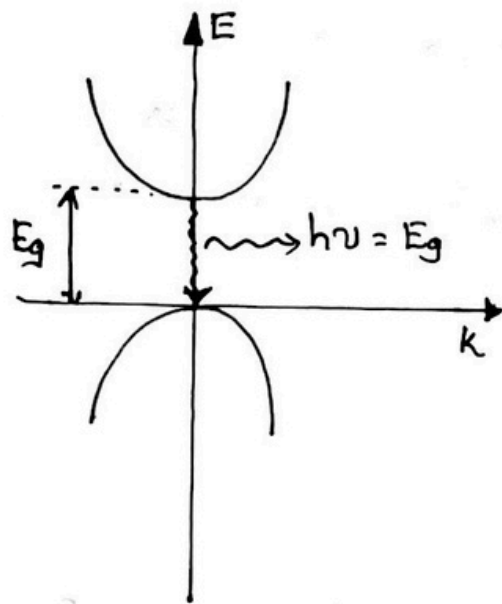
Fig. Donation of electrons from donor level to conduction band.

P - Type Semiconductors:

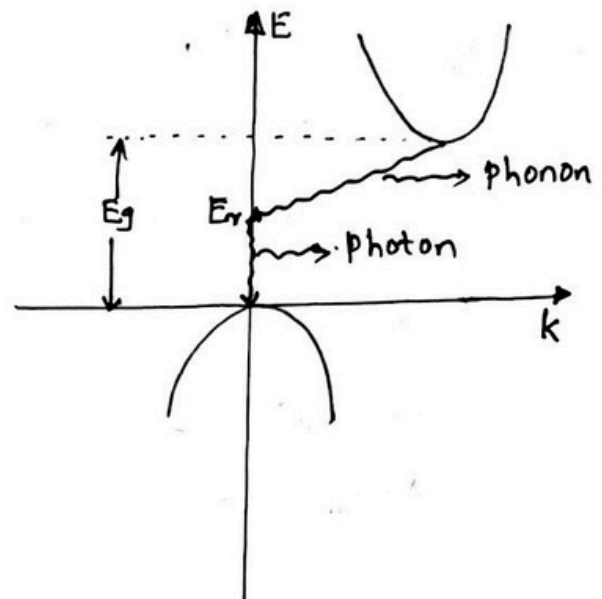


Direct and indirect semiconductors

Direct SC



Indirect SC



Direct SC

- Minimum energy of $\bar{e}$ in the CB equals maximum energy of $\bar{e}$ in the VB.
- ie, They have same momentum value or k value ($k=0$) for $\bar{e}$ s in the CB and holes in the VB.
- $\bar{e}$ s in the CB can easily fall to an ~~energy~~ empty state in the VB giving off the energy ~~as~~ difference E_g as Photon of light w/o change in k or p value.

eg:- GaAs.

Applications: LEDs, Lasers

Indirect SC

- $\bar{e}$ s in CB_{min} and VB_{max} have different values of k . and thus momentum values are also different ($p = \hbar k$)
- So $\bar{e}$ cannot recombine directly with a hole in the VB by releasing a photon.
- So $\bar{e}$ in CB_{max} undergoes a momentum change and

energy change and come to an intermediate position between CB_{min} and VB_{max} ~~at~~ releasing a phonon.

→ This state is called recombination level (E_r) or E_t or defect level, from where it falls ~~at~~ to the VB_{max} giving off energy difference as a photon of light.

eg: Si, Ge

Applications: Solar cells.

Relation between Energy (E) and Momentum (P)

$$E = \frac{1}{2}mv^2 \quad \text{--- (1)}$$

We know, $P = mv \quad \text{--- (2)}$

$$\text{(1)} \Rightarrow E = \frac{1}{2} \frac{P^2}{m}$$

From De Broglie eqn, $\lambda = \frac{h}{p}$

$$P = \frac{h}{\lambda} = \frac{h/2\pi}{\lambda/2\pi}$$

$$\underline{P = \hbar k} \quad \text{where } k = \frac{2\pi}{\lambda}$$

P : angular momentum, k : wave vector

$$E = \frac{P^2}{2m} = \frac{\hbar^2 k^2}{2m}$$

$E \propto k^2$ ie, E vs $k(p)$ is a parabolic curve, hence both CB and VB are seen to be parabolic.

Effective Mass

→ The movement of an electron in a lattice is different from that of an electron in free space.

In a crystal lattice there are lots of internal forces due to positively charged ions/protons and negatively charged ions (electrons) which influences $\bar{e}$ motion in the lattice.

→ So the mass of $\bar{e}$ to be used cannot be that of individual $\bar{e}$. It needs to be w.r.t the effect of internal forces.

$$p = mv$$

$$\begin{aligned} E &= \frac{1}{2}mv^2 = \frac{1}{2} \frac{p^2}{m} \\ &= \frac{(\hbar k)^2}{2m} = \frac{\hbar^2 k^2}{2m} \quad \text{--- (1)} \end{aligned}$$

Here the mass considered should be the effective mass of $\bar{e}$. (m_n^*)

$$\begin{aligned} \text{(1)} \Rightarrow E &= \frac{\hbar^2 k^2}{2m} \\ \frac{dE}{dk} &= \frac{\hbar^2 \cdot 2k}{2m} = \frac{\hbar^2 k}{m} = \frac{p\hbar}{m} \\ \frac{dE}{dk} \cdot \frac{1}{\hbar} &= \frac{p}{m} = v \quad \text{--- (2)} \end{aligned}$$

Thus first derivative of E with respect to k is related to velocity of the particle (v)

$$\begin{aligned} \frac{d^2 E}{dk^2} &= \frac{\hbar^2}{m} \\ \frac{1}{\hbar^2} \cdot \frac{d^2 E}{dk^2} &= \frac{1}{m} \quad \text{--- (3)} \end{aligned}$$

second derivative of E with respect to k is inversely proportional to mass of the particle.

$$\textcircled{3} \Rightarrow m^* = \frac{\hbar^2}{d^2E/dk^2}$$

Equilibrium and steady state conditions:

→ Equilibrium means thermal equilibrium. The solid has one well defined temperature and a constant Fermi energy.

→ Steady state means that the properties of the system do not change with time.

Non equilibrium states can be steady states if there is a source of energy to maintain the non equilibrium condition. Without the source of energy, the system would quickly settle into an equilibrium state.

→ A p-n junction with no voltage applied is in an equilibrium state.

→ The p-n junction diode with a voltage applied across it is in a non equilibrium, but steady state.

Equilibrium: Condition of no external excitation (except temperature) and no net motion of charge.

Steady state: Non equilibrium state in which all processes are constant and are balanced by opposing processes.

In order to understand the electrical properties of a semiconductor, it is necessary to understand the number of charge carriers/cm³ in the material, which is called carrier concentration.

- To obtain the equations for carrier concentration, the distribution of carriers over the available energy states needs to be studied. This is done by using statistical methods.
- ie, studying the property/~~of~~ a behaviour of a group as a whole instead of as an individual particle
- This is done by using Fermi Dirac Distribution Function (FDD).
- It gives the distribution of E_s over a range of allowed energy levels at thermal equilibrium.

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}} \quad \text{--- (1)}$$

where

$f(E)$: Fermi Dirac Distribution Function

K : Boltzmann's constant $8.62 \times 10^{-5} \text{ eV/K}$

T : Absolute Temperature $1.38 \times 10^{-23} \text{ J/K}$

E_F : Fermi energy level / Fermi level

$f(E)$ gives the probability that the energy level 'E' is filled by electrons at absolute temperature under thermal equilibrium.

At 0K,

⇒ when $E > E_F$, $f(E) = \frac{1}{1 + e^{(E-E_F)/K \times 0}} = \frac{1}{1 + e^{\infty}} = \frac{1}{\infty} = 0$

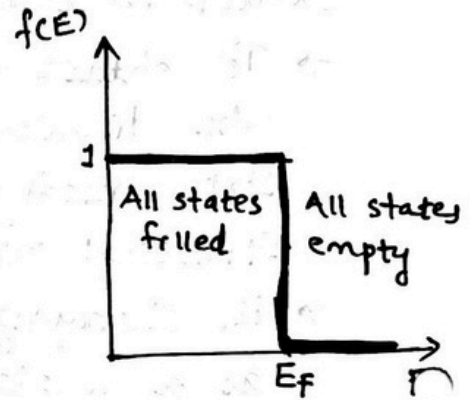
When $E < E_F$,

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + \frac{1}{e^{\infty}}} = \frac{1}{1 + 0} = \frac{1}{1} = 1$$

So $f(E > E_F) = 0$
 $f(E < E_F) = 1$ } at 0K

When $E = E_F$,

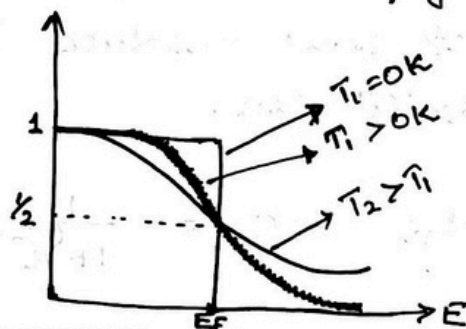
$$\Rightarrow f(E = E_F) = \frac{1}{1 + e^{(E_F - E_F)/kT}} = \frac{1}{1 + e^{0/kT}} = \frac{1}{1 + 1} = \frac{1}{2}$$



Thus Fermi level (E_F) is the energy level at which probability of half being occupied by an electron.
 or the energy level at which the probability of finding the electron is 50%.

At 0K, curve takes rectangular distribution implies that, every available states up to E_F is filled with electrons and all the states above E_F is empty.

→ At some temperature above 0K (say T_1), there is some probability of $f(E)$ that states above E_F are filled and a corresponding probability $[1 - f(E)]$ that states below E_F are empty.

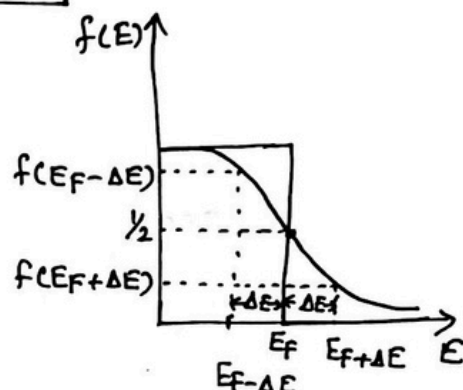


→ Probability $f(E_F + \Delta E)$ that a state ΔE above E_F is filled is the same as the probability $[1 - f(E_F - \Delta E)]$ that a state ΔE below E_F is empty.

Prove that $f(E_F + \Delta E) = 1 - f(E_F - \Delta E)$

Proof:

$$\begin{aligned} f(E_F + \Delta E) &= \frac{1}{1 + e^{(E_F + \Delta E - E_F)/KT}} \\ &= \frac{1}{1 + e^{\Delta E/KT}} \end{aligned}$$



Probability ~~of~~ that a state ΔE below E_F is filled,

$$f(E_F - \Delta E)$$

So probability that a state ΔE below E_F is empty

$$1 - f(E_F - \Delta E).$$

$$\begin{aligned} f(E_F - \Delta E) &= \frac{1}{1 + e^{(E_F - \Delta E - E_F)/KT}} \\ &= \frac{1}{1 + e^{-\Delta E/KT}} \end{aligned}$$

$$\begin{aligned} 1 - f(E_F - \Delta E) &= 1 - \frac{1}{1 + e^{-\Delta E/KT}} \\ &= \frac{1 + e^{-\Delta E/KT} - 1}{1 + e^{-\Delta E/KT}} = \frac{e^{-\Delta E/KT}}{1 + e^{-\Delta E/KT}} \end{aligned}$$

Divide Nr and Dr by $e^{\Delta E/KT}$

$$\Rightarrow \frac{e^{-\Delta E/KT} / e^{\Delta E/KT}}{\frac{1}{e^{\Delta E/KT}} + \frac{e^{-\Delta E/KT}}{e^{\Delta E/KT}}}$$

$$\begin{aligned} &= \frac{1}{\frac{1}{e^{\Delta E/KT}} + 1} = \frac{1}{\frac{1}{e^{\Delta E/KT}} + 1} \\ &= \frac{1}{1 + e^{-\Delta E/KT}} \\ &= \underline{\underline{f(E_F + \Delta E)}} \end{aligned}$$

Fermi Dirac Distribution applied to intrinsic and Extrinsic semiconductors

Intrinsic SC

In intrinsic SC,

No. of $\bar{e}$ s = ~~No. of $\bar{e}$ s~~ = No of holes

$$n_0 = p_0$$

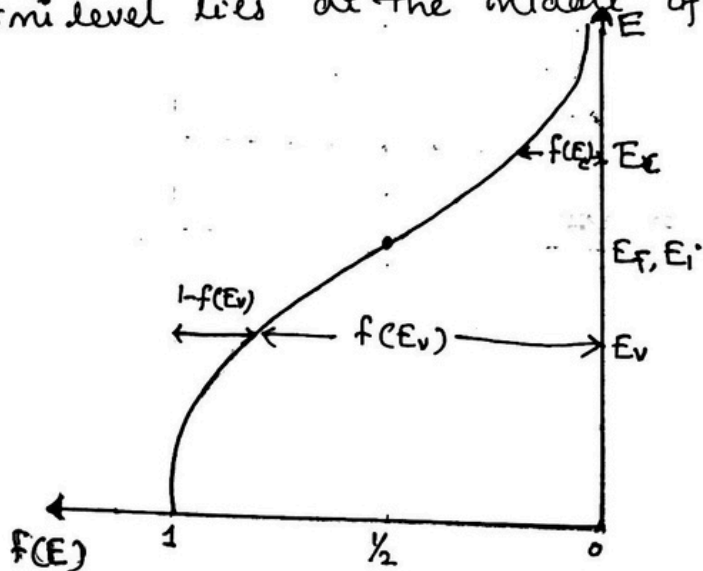
In this case, there is equal no. of states at the edges of CB and VB. i.e., there is equal chances of finding an $\bar{e}$ at CB edge as there is of finding a hole at the VB edge.

Probability of occupancy in CB = $f(E_c)$
" " vacancy in VB = $1 - f(E_v)$

$$f(E_c) = 1 - f(E_v)$$

i.e., Concentration of holes in VB = concentration of $\bar{e}$ s in CB

So Fermi level lies at the middle of bandgap.



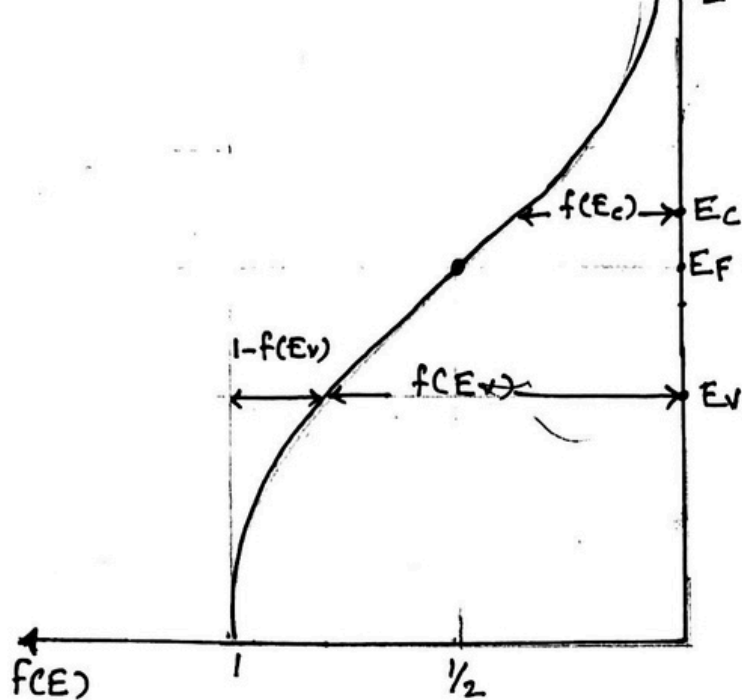
$$f(E_c) = 1 - f(E_v)$$

Extrinsic SC.

a) N-type SC

In N-type SC, no. of $\bar{e}$ s in CB is more compared to hole concentration in VB.

So $f(E_c) \gg 1 - f(E_v)$

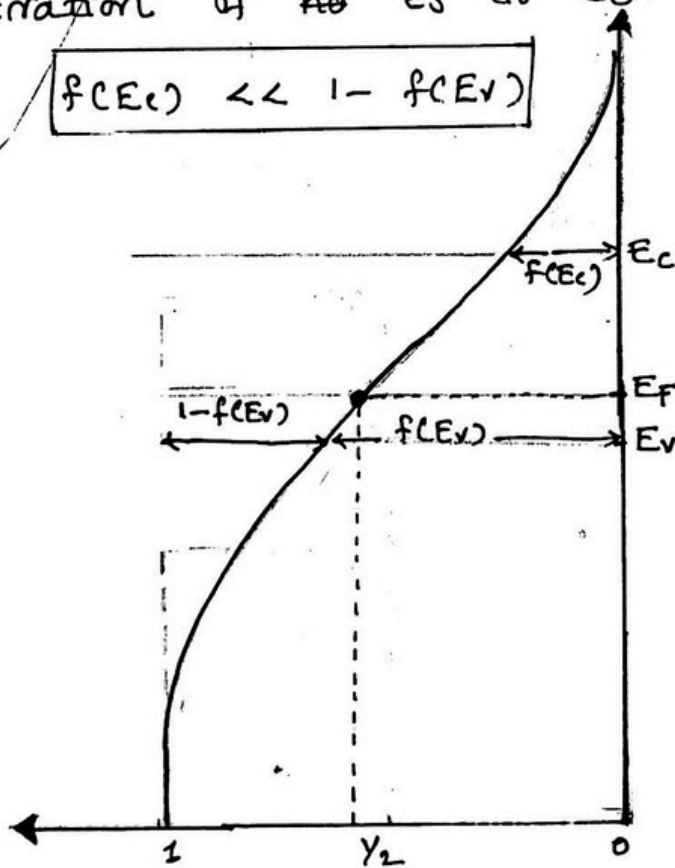


$$f(E_c) \gg 1 - f(E_v)$$

(b) P type SC

Concentration of holes in v_b is greater than concentration of e_s in c_b .

$$f(E_c) < 1 - f(E_v)$$



$$f(E_c) < 1 - f(E_v)$$

Electron and Hole concentrations at equilibrium (n_0, p_0)

The Fermi Dirac Distribution can be used to calculate the concentrations of electrons & holes in a sc, if we know the densities of available states in CB and VB.

$$\text{ie, } n_0 = \int_{E_c}^{\infty} f(E) N(E) dE \quad \text{--- (1)}$$

where $N(E)dE$ is the density of states per cm^3 in the energy range dE .
(DOS)

So number of es per unit volume in the energy range dE is the product of DOS and the probability of occupancy.
Let N_c be the effective density of states for the distributed electron states in the CB edge.

Then

$$n_0 = N_c \cdot f(E_c) \quad \text{--- (2)}$$

$$\text{We know, } f(E_c) = \frac{1}{1 + e^{(E_c - E_f)/KT}} \quad \text{--- (3)}$$

$$\text{but } E_c - E_f \gg KT$$

$$\text{(3)} \Rightarrow f(E_c) = \frac{1}{e^{(E_c - E_f)/KT}} = e^{-(E_c - E_f)/KT}$$

$$f(E_c) = e^{-(E_c - E_f)/KT} \quad \text{--- (4)}$$

$$\text{(2)} \Rightarrow \boxed{n_0 = N_c \cdot e^{-(E_c - E_f)/KT}} \quad \text{--- (5)}$$

$$\text{where } N_c = 2 \left(\frac{2\pi m_n^* KT}{h^2} \right)^{3/2}$$

Concentration of holes in VB is

$$p_0 = N_v [1 - f(E_v)] \quad \text{--- (6)}$$

where N_v is the effective DOS in the VB.

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$$\begin{aligned}
 1 - f(E_v) &= 1 - \frac{1}{1 + e^{(E_v - E_F)/KT}} \\
 &= \frac{e^{(E_v - E_F)/KT}}{1 + e^{(E_v - E_F)/KT}} \\
 &= \frac{e^{(E_v - E_F)/KT}}{e^{(E_v - E_F)/KT} + 1}
 \end{aligned}$$

Divide Numerator & Denominator with $e^{(E_v - E_F)/KT}$

$$1 - f(E_v) = \frac{1}{e^{- (E_v - E_F)/KT} + 1}$$

But $E_F - E_v \gg KT$

$$1 - f(E_v) = \frac{1}{e^{- (E_v - E_F)/KT}} = e^{(E_v - E_F)/KT}$$

$$1 - f(E_v) = e^{-(E_F - E_v)/KT} \quad \text{--- (7)}$$

$$\textcircled{6} \Rightarrow \boxed{p_0 = N_v \cdot e^{-(E_F - E_v)/KT}} \quad \text{--- (8)}$$

$$\text{where } N_v = 2 \left(\frac{2\pi m_p^* KT}{h^2} \right)^{3/2}$$

Multiply $\textcircled{5}$ and $\textcircled{8}$

$$n_0 p_0 = N_c \cdot e^{-(E_c - E_F)/KT} \cdot N_v \cdot e^{-(E_F - E_v)/KT}$$

$$n_0 p_0 = N_c \cdot N_v \cdot e^{-(E_c - E_v)/KT} \quad \text{--- (9)}$$

$$n_0 p_0 = N_c \cdot N_v \cdot e^{-E_g/KT} \quad \text{--- (10)} \quad \because E_c - E_v = E_g$$

For intrinsic material, E_F lies at E_i

$$\textcircled{5} \Rightarrow n_i = N_c \cdot e^{-(E_c - E_i)/KT} \quad \text{--- (11)}$$

$$\textcircled{8} \Rightarrow P_i = N_v \cdot e^{-\frac{(E_i - E_v)}{kT}} \quad \text{--- (12)}$$

Multiply $\textcircled{11}$ & $\textcircled{12}$,

$$n_i \cdot P_i = N_c \cdot e^{-\frac{(E_c - E_i)}{kT}} \cdot N_v \cdot e^{-\frac{(E_i - E_v)}{kT}}$$

$$n_i \cdot P_i = N_c \cdot N_v \cdot e^{-\frac{(E_c - E_v)}{kT}} \quad \text{--- (13)}$$

$$n_i \cdot P_i = n_i^2 \quad \text{Since } n_i = P_i$$

$$\textcircled{13} \Rightarrow n_i^2 = N_c \cdot N_v \cdot e^{-\frac{(E_c - E_v)}{kT}}$$

$$n_i^2 = N_c \cdot N_v \cdot e^{-\frac{E_g}{kT}} \quad \text{--- (14)}$$

$$n_i = \sqrt{N_c \cdot N_v} \cdot e^{-\frac{E_g}{2kT}} \quad \text{--- (15)}$$

Compare $\textcircled{10}$ and $\textcircled{14}$, gives

$$n_0 \cdot p_0 = n_i^2 \quad \text{--- Mass Action Law (16)}$$

compare $\textcircled{11}$ and $\textcircled{15}$

$$\text{If } N_c = N_v \text{ in (15), } E_c - E_i = \frac{E_g}{2} \quad \text{--- (17a)}$$

ie, Intrinsic level E_i is at the middle of bandgap.

$$\text{or } E_i - E_v = \frac{E_g}{2} \quad \text{--- (17b)}$$

n_0, p_0 in terms of n_i, p_i

$$\textcircled{5} \Rightarrow N_c = \frac{n_0}{e^{-\frac{(E_c - E_F)}{kT}}}$$

$$\textcircled{4} \Rightarrow N_c = \frac{n_i}{e^{-\frac{(E_c - E_i)}{kT}}}$$

Equating,

$$\Rightarrow \frac{n_0}{e^{-\frac{(E_c - E_F)}{kT}}} = \frac{n_i}{e^{-\frac{(E_c - E_i)}{kT}}} \quad \text{--- (18)}$$

$$\Rightarrow n_0 = n_i \cdot e^{\frac{(E_F - E_i)}{kT}}$$

For P_0 ,

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$$(8) \Rightarrow N_V = \frac{P_0}{e^{-(E_F - E_V)/KT}}$$

$$(12) \Rightarrow N_V = \frac{P_i}{e^{-(E_i - E_V)/KT}}$$

Equating,

$$\frac{P_0}{e^{-(E_F - E_V)/KT}} = \frac{P_i}{e^{-(E_i - E_V)/KT}}$$

$$P_0 = P_i \cdot e^{(E_i - E_F)/KT} \quad \text{--- (19)}$$

Note :

In (18), when $E_F = E_i$, $n_0 = n_i$ ^{For} Intrinsic S.C E_F lies at E_i .

As E_F moves away from E_i towards CB,

$E_F - E_i \uparrow$ so $n_0 \uparrow$ exponentially.

Why P_0 varies from P_i or n_i to higher values as E_F moves away from E_i towards VB.

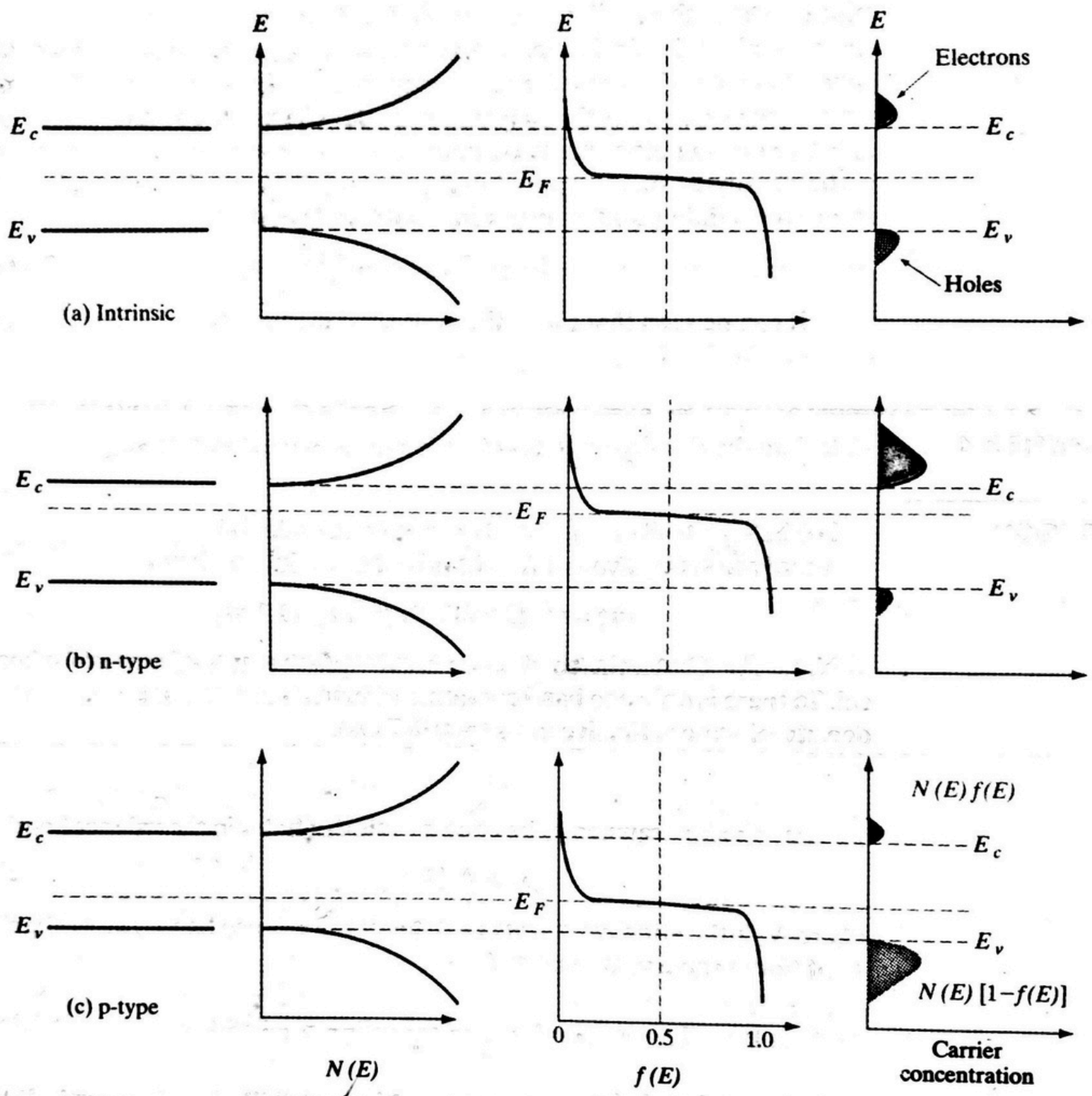


Figure 3-16

Schematic band diagram, density of states, Fermi-Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium.

Excess carriers in semiconductors:

Charge carriers in semiconductors which are in excess of their thermal equilibrium value are called excess carriers. They can be generated by

1. Optical excitation
2. Electron bombardment

Consider a semiconductor. A photon of energy $h\nu$ is incident on it. If $h\nu \geq E_g$, the photon is absorbed in the sc, causing excitation of e^- s from VB to CB. This results in the generation of EHPs. These EHPs in excess of their thermal equilibrium values are called excess carriers.

Optically generated $\bar{e}$ and hole concentrations are equal.

$n = n_0 + \delta n$
$p = p_0 + \delta p$
$\delta n = \delta p$

n : Total $\bar{e}$ concentration

n_0 : equilibrium $\bar{e}$ concentration

δn : excess $\bar{e}$ concentration

p : Total hole concentration

p_0 : equilibrium hole concentration

δp : excess hole concentration

Low level injection:

If excess carrier concentration is small compared to the equilibrium majority carrier concentration.

$$\text{ie, } \underline{\delta n, \delta p \ll n_0} \quad (n \text{ type})$$

$$\underline{\delta n, \delta p \ll p_0} \quad (p \text{ type})$$

High level injection:

$$\delta n, \delta p \gg n_0 \quad (n \text{ type})$$

$$\delta n, \delta p \gg p_0 \quad (p \text{ type})$$

Excess carrier lifetime : Direct recombination

In direct recombination, $\bar{e}$ from CB falls into a vacant state (hole) in the VB and the difference in energy is released as a photon.

Rate of decay of $\bar{e}$ at any time t is proportional to the product of number of $\bar{e}$ s and number of holes remaining at time t .

Net rate of change in $\bar{e}$ concentration,

$$\frac{d n(t)}{dt} = \alpha_g \cdot n_i^2 - \alpha_r n(t) \cdot p(t) \quad \text{--- (1)}$$

(Thermal generation rate - Recombination rate)

But we have

$$n = n_0 + \delta n(t) \quad \text{--- (a)}$$

$$p = p_0 + \delta p(t) \quad \text{--- (b)}$$

Substitute 2 (a) & (b) in (1),

$$\frac{d(n_0 + \delta n)}{dt} = \alpha_g n_i^2 - \alpha_r (n_0 + \delta n) (p_0 + \delta p)$$

$$\begin{aligned} \frac{d n_0}{dt} + \frac{d \delta n(t)}{dt} &= \alpha_g n_i^2 - \alpha_r [n_0 + \delta n(t)] [p_0 + \delta p(t)] \\ &= \alpha_g n_i^2 - \alpha_r [n_0 p_0 + p_0 \delta n(t) + n_0 \delta p(t) + \delta n(t) \cdot \delta p(t)] \end{aligned}$$

$$\text{But } \delta n(t) = \delta p(t)$$

$$= \alpha_r \cdot n_i^2 - \alpha_r [n_0 p_0 + p_0 \delta n(t) + n_0 \delta n(t) + \delta n^2(t)]$$

$$= \alpha_r n_i^2 - \alpha_r [n_0 p_0 + \delta n(t) [n_0 + p_0] + \delta n^2(t)] \quad \text{--- (3)}$$

$$\alpha_r n_0 p_0 = \alpha_r \cdot n_i^2$$

② $\frac{d}{dt} n_0 = 0 \quad \therefore n_0 \text{ \& } p_0 \text{ are equilibrium values which are constant.}$

$$\textcircled{3} \Rightarrow \frac{d}{dt} \delta n(t) = \alpha_r n_i^2 - \alpha_r n_i^2 - \alpha_r \cdot (n_0 + p_0) \delta n(t) - \alpha_r \cdot \delta n^2(t)$$

$$\frac{d}{dt} \delta n(t) = -\alpha_r \cdot (n_0 + p_0) \delta n(t) - \alpha_r \delta n^2(t) \quad \text{--- (4)}$$

Consider low level injection,

$\delta n(t)$ is small, so $\delta n^2(t)$ is also small. neglect

For P type, $p_0 \gg n_0$, neglect n_0 .

$$\textcircled{4} \Rightarrow \frac{d}{dt} \delta n(t) = -\alpha_r \cdot p_0 \cdot \delta n(t) \quad \text{--- (5)}$$

Soln is $\delta n(t) = \Delta n \cdot e^{-\alpha_r \cdot p_0 \delta n(t)} = \Delta n \cdot e^{-t/\tau_n}$

$$\boxed{\delta n(t) = \Delta n \cdot e^{-t/\tau_n}} \quad \text{--- (6)}$$

Δn : Initial excess carrier concentration ($t=0$)

$$\boxed{\tau_n = \frac{1}{\alpha_r \cdot p_0}} = \text{Minority carrier life time}$$

or
Recombination life time in P type

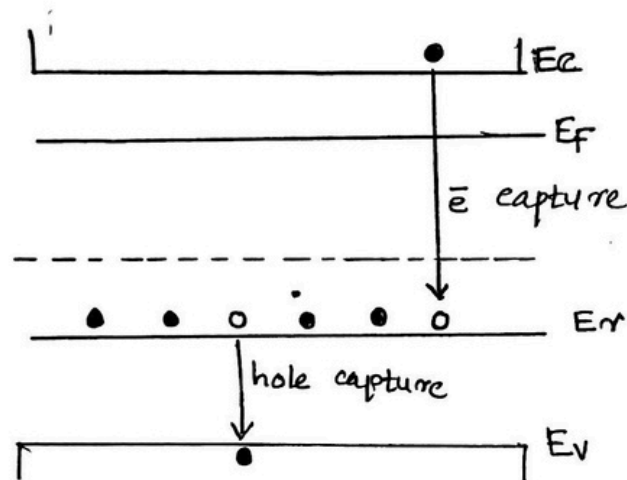
$$\boxed{\tau_p = \frac{1}{\alpha_r \cdot n_0}}$$

In general, $\boxed{\tau_n = \frac{1}{\alpha_r (n_0 + p_0)}}$

Indirect recombination; Trapping

In indirect recombination semiconductors, recombination occurs through recombination levels or centres within the bandgap.

Any impurity or lattice defect can serve as recombination centre, if it is capable of receiving carriers of one type and subsequently capturing the opposite type of carriers, thereby annihilating the pair.



The recombination level E_r is filled with e^- s. When excess electrons and holes are created, each electron-hole pair recombines at E_r in two steps.

1. Hole capture:

Falling of an electron from E_r to the VB leaving behind an empty state in the recombination level E_r creating heat energy.

2. Electron capture

When e^- in CB falls to an empty state at E_r , it is called e^- capture.

Trapping

Recombination can be delayed if the captured carrier is thermally reexcited back to its original band before the capture of opposite type of carrier i.e. If electron capture does not follow immediately after hole capture, the hole is thermally reexcited to the VB.

→ This process delays recombination because a hole capture needs to occur again before the recombination can be completed.

→ When a carrier is trapped temporarily at a centre and reexcited without recombination, that process is called temporary trapping.

Steady state Carrier Generation:

At equilibrium,

Generation rate = Recombination rate

$$g(t) = \alpha_r n_i^2 = \alpha_r n_0 P_0 \quad \text{--- (1)}$$

When optical excitation is considered,

$$g(t) + g_{op} = \alpha_r n p \quad \text{--- (2)}$$

$$= \alpha_r (n_0 + \delta n) (P_0 + \delta p)$$

$$= \alpha_r n_0 P_0 + \alpha_r n_0 \delta p + \alpha_r P_0 \delta n + \alpha_r \delta p \delta n$$

$$\therefore \delta n = \delta p$$

$$g(t) + g_{op} = \alpha_r n_0 P_0 + \alpha_r n_0 \delta n + \alpha_r P_0 \delta n + \alpha_r \delta n^2$$

$$= \alpha_r n_0 P_0 + \alpha_r (n_0 + P_0) \delta n + \alpha_r \delta n^2$$

δn is small, neglect δn^2

$$g(t) + g_{op} = \underbrace{\alpha_f n_0 p_0}_{g(t)} + \alpha_f (n_0 + p_0) \delta n \quad \text{--- (3)}$$

$$\text{So } g_{op} = \alpha_f (n_0 + p_0) \delta n$$

$$g_{op} = \frac{\delta n}{\tau_n}$$

$$\boxed{\delta n = \delta p = g_{op} \cdot \tau_n}$$

Quasi Fermi levels, F_n, F_p

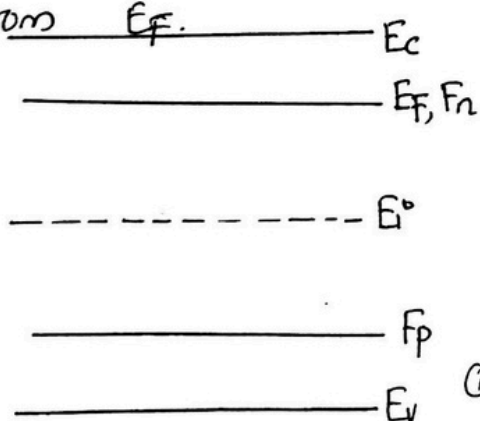
When excess carriers are present, equilibrium Fermi level E_f is meaningless. It does not represent carrier concentration. So separate quasi Fermi levels are defined.

F_n = Excess Fermi level for e^- s

F_p : Excess Fermi level for holes

If n type material is considered, it has Fermi level E_f near the CB.

If excess carriers are added, $\delta n = \delta p$. Thus F_n is close to E_f (or similar to E_f), but F_p is far away from E_f .



When excess carriers are present, the deviations F_n & F_p from E_f indicates how far e^- s & holes are from equilibrium values of n_0 & p_0 .

$$n = n_i e^{(F_n - E_i)/kT} \quad \text{--- (1)}$$

$$p = p_i e^{(E_i - F_p)/kT} \quad \text{--- (2)}$$

$$\textcircled{1} \times \textcircled{2}, \quad n p = n_i p_i e^{(F_n - F_p)/kT}$$

$$\boxed{n p = n_i^2 e^{(F_n - F_p)/kT}} \quad \text{since } n_i = p_i$$

Module 2: Carrier Transport in Semiconductors

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Basic Transport mechanisms are

1. Drift.
2. Diffusion

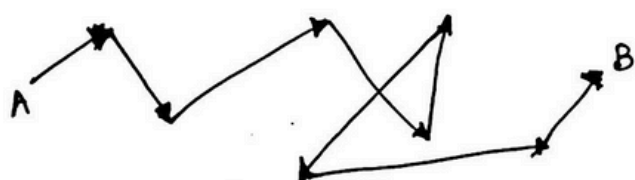
Drift:

An Electric field applied to a sc will produce a force on electrons and holes so that they will experience a net acceleration and a net movement, provided there are available energy states in the CB and VB. This net movement of charge due to electric field is called drift. The net drift of charge give rise to drift current.

Drift current Density (J_{drift})

Conductivity (σ), Mobility (μ)

→ The charge carriers in a solid are in constant motion.



Thermal motion of e^- in a solid.

The e^- at point 'A' need not return back to its initial point.

* An Electric field E_x is applied in x direction ie, E_{sc}

Force experienced by each e^- is

$$F_x = -q E_{\text{sc}} \quad \text{--- (1)}$$

If we have 'n' number of e^- , net force experienced by a group of e^- is

$$F_x = -n q E_{\text{sc}} \quad \text{--- (2)}$$

If the P_x is the total momentum of a group of 'n' e^-/cm^3 along x direction. Then

$$F_x = \frac{dP_x}{dt} \quad \text{--- (3)}$$

From ② & ③, we can write

$$\left. \frac{dP_x}{dt} \right|_{\text{field}} = -nqE_x \quad \text{--- ④}$$

This gives the continuous acceleration of $\bar{e}$ s in the $-x$ direction.

→ The force due to electric field is balanced in steady state by the decelerating force of the collision process.

→ To evaluate the change in momentum due to collisions, collision probabilities must be investigated closely.

→ The probability of collision for an $\bar{e}$ in any time interval dt is $\frac{dt}{\bar{\tau}}$ where $\bar{\tau}$: mean free time b/w collision or scattering events

So differential change in momentum (P_x) due to collision in time dt is

$$dP_x = -P_x \cdot \frac{dt}{\bar{\tau}} \quad \text{--- ⑤}$$

The rate of change of P_x due to the decelerating effect of collisions is

$$\left. \frac{dP_x}{dt} \right|_{\text{collision}} = -\frac{P_x}{\bar{\tau}} \quad \text{--- ⑥}$$

At steady state, sum of accelerating and decelerating effects must be zero.

$$\text{④} + \text{⑥}, \quad \left. \frac{dP_x}{dt} \right|_{\text{field}} + \left. \frac{dP_x}{dt} \right|_{\text{collision}} = -\frac{P_x}{\bar{\tau}} + nqE_x = 0$$

$$\text{ie, } -\frac{P_x}{\bar{\tau}} + nqE_x = 0$$

$$-\frac{P_x}{\bar{\tau}} = -nqE_x$$

$$P_x = -nq\bar{\tau}E_x \quad \text{--- ⑦}$$

The average momentum per $\bar{e}$ is

$$\langle P_x \rangle = \frac{P_x}{n} = -q \bar{E} \epsilon_x \quad (8)$$

$\langle \rangle$ denotes average over entire group of $\bar{e}$ s

$$\langle v_x \rangle = \frac{\langle P_x \rangle}{m_n^*} = -\frac{q \bar{E}}{m_n^*} \epsilon_x \quad (9)$$

Drift velocity.

We know,

$$I_x = -nq \langle v_x \rangle \quad (10) \Rightarrow I = nqv.$$

Substitute (9) in (10)

$$I_x = -nq \cdot -\frac{q \bar{E}}{m_n^*} \epsilon_x$$

$$I_x = \frac{nq^2 \bar{E}}{m_n^*} \epsilon_x \quad (11)$$

$$\Rightarrow I_x = \sigma \epsilon_{px} \quad (12) \Rightarrow I_x \propto \text{Electric field } \epsilon_x$$

where

$$\sigma = \frac{nq^2 \bar{E}}{m_n^*} \quad (13)$$

Conductivity.

We have,

$$\sigma = nq \mu_n \quad (14)$$

where

$$\mu_n = \frac{q \bar{E}}{m_n^*} \quad (15)$$

Electron mobility: describes the ease with

electrons drift in the material.

(9) can be written as

$$\langle v_x \rangle = -\frac{q \bar{E}}{m_n^*} \epsilon_x = -\mu_n \cdot \epsilon_x$$

$$\mu_n = -\frac{\langle v_x \rangle}{\epsilon_x} \quad (16)$$

(12) $\Rightarrow$

$$I_x = \sigma \epsilon_{px} = nq \mu_n \epsilon_{px} \quad (17)$$

This derivation is based on the assumption that current is carried primarily by $\bar{e}$ s.

If both $\bar{e}$ s and holes participate in current flow,

$$I_{\text{drift}} = I_x = nq \mu_n \epsilon_x + p q \mu_p \epsilon_x \quad (18)$$

$$I_x = I_{\text{drift}} = I_x A = A [nq \mu_n \epsilon_x + p q \mu_p \epsilon_x]$$

Diffusion of Carriers.

When excess carriers are created non uniformly in a sc, the $\bar{e}$ and hole concentration vary with position in the sample.

→ Now there is a region of higher concentration and a region of lower concentration.

→ Hence carriers move from a high concentration to a low concentration. This motion of charge carriers is called ~~charge carriers~~ Diffusion. and the current developed is called diffusion current.

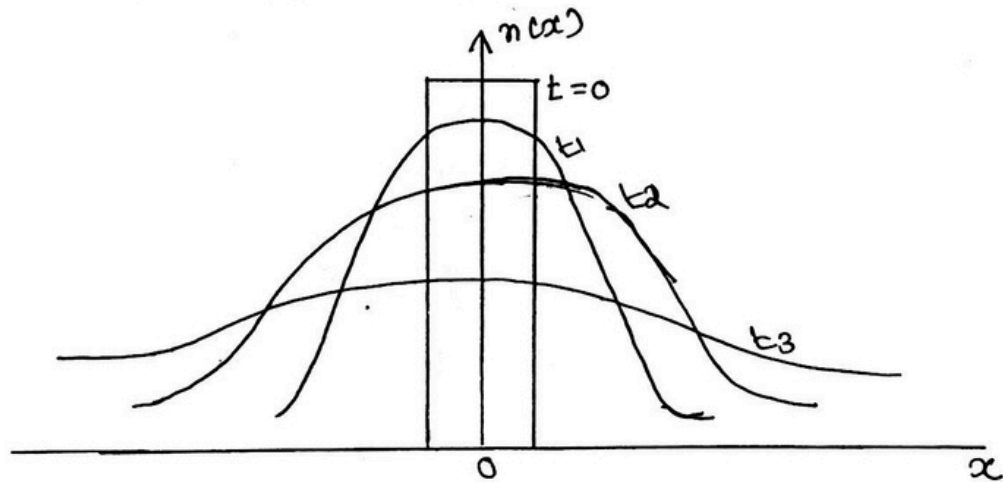


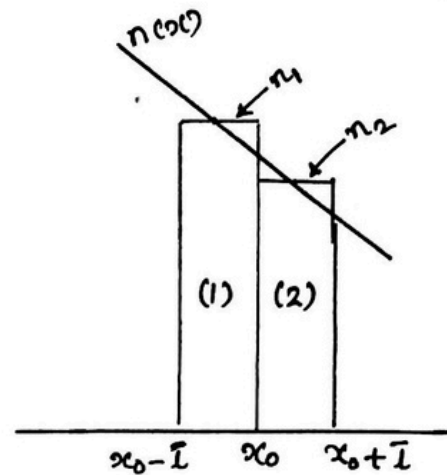
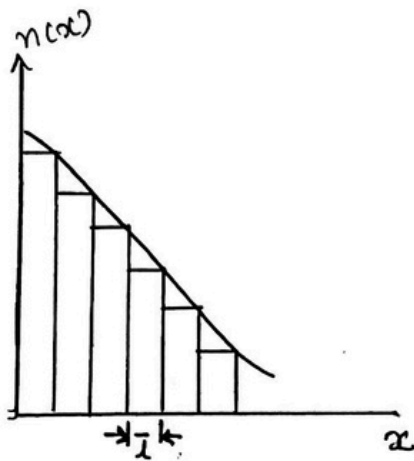
fig: Spreading of pulse of $\bar{e}$ s by diffusion

- A pulse of excess $\bar{e}$ s are concentrated at $x=0$ at time $t=0$ will spread out in time as in fig.
- At $t=0$, excess $\bar{e}$ s are concentrated at $x=0$. As time $\uparrow$ s, carriers spread out across x .
- Finally $\bar{e}$ s would have diffused from regions of higher to lower concentration. Finally $n(x)$ becomes uniform throughout.

* Rate of $\bar{e}$ diffusion: Diffusion Current:

Consider distribution of $\bar{e}$ s as $n(x)$ and x is divided into segments of width $\bar{l}$ (mean free path)

$n(x)$ is evaluated at the centre of each segment.



- Since the movement of $\bar{e}$ s is random, the $\bar{e}$ s in a segment can move either to the right or to the left.
- In one mean free time, $\frac{1}{2}$ of the $\bar{e}$ s from segment 1 crosses x_0 and moves in to segment 2. and $\frac{1}{2}$ of the $\bar{e}$ s in segment 2 will cross x_0 and move to segment 1.

$$\bar{e} \text{ concentration in seg 1} = n_1$$

$$\bar{e} \text{ concentration in seg 2} = n_2$$

$$\text{Concn} = \frac{\text{number}}{\text{volume}}$$

$$\text{Total no. of } \bar{e}\text{s in seg 1} = n_1 \cdot \bar{\Delta}A$$

$$\text{" " } \bar{e}\text{s in seg 2} : n_2 \bar{\Delta}A$$

$$\text{Net no. of } \bar{e}\text{s crossing from left to right in one mean free time } (\bar{\tau}) = \frac{1}{2} [n_1 \bar{\Delta}A - n_2 \bar{\Delta}A]$$

$$= \frac{1}{2} [n_1 - n_2] \bar{\Delta}A \quad \text{--- ①}$$

Diffusion current density = $\bar{e}$ flux density $\times$ charge.

→ Electron flux density, $\phi_n(x)$ is the rate of $\bar{e}$ flow per unit area.

$$\phi_n(x) = \frac{\text{Net no. of } \bar{e}\text{s moving in } x \text{ direction}}{\text{Time} \times \text{Area}}$$

$$\phi_n(x) = \frac{\frac{1}{2} \bar{L} A (n_1 - n_2)}{\bar{L} A} \quad \text{--- (2)}$$

$$\phi_n(x) = \frac{\bar{L} (n_1 - n_2)}{2 \bar{L}} \quad \text{--- (3)}$$

Difference in $\bar{n}$ concentration, $n_1 - n_2$ can be written as

$$n_1 - n_2 = \frac{n(x) - n(x + \Delta x) \cdot \bar{L}}{\Delta x} \quad \text{--- (4)} \quad \Delta x = \bar{L}$$

Substitute (4) in (3)

$$\phi_n(x) = \frac{\bar{L}}{2 \bar{L}} \left[\lim_{\Delta x \rightarrow 0} \frac{n(x) - n(x + \Delta x)}{\Delta x} \right] \bar{L}$$

$$\phi_n(x) = -\frac{\bar{L}^2}{2 \bar{L}} \frac{d n(x)}{d x} \quad \text{--- (5)}$$

$$\left[\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{d f(x)}{d x} \right]$$

$\frac{1}{2} \frac{\bar{L}^2}{\bar{L}} = \text{Electron diffusion coefficient } (D_n) \text{ (cm}^2/\text{s)}$

$$(5) \Rightarrow \phi_n(x) = -D_n \cdot \frac{d n(x)}{d x} \quad \text{--- (6)}$$

Electron current density = flux density $\times$ charge.

$$J_n(x) = \cancel{L} \phi_n(x) \cdot (-q)$$

$J_n(x) = q D_n \cdot \frac{d n(x)}{d x}$
$J_p(x) = -q D_p \cdot \frac{d p(x)}{d x}$

So $J_n(x) = A \cdot q D_n \cdot \frac{d n(x)}{d x}$

$J_p(x) = -A q D_p \cdot \frac{d p(x)}{d x}$

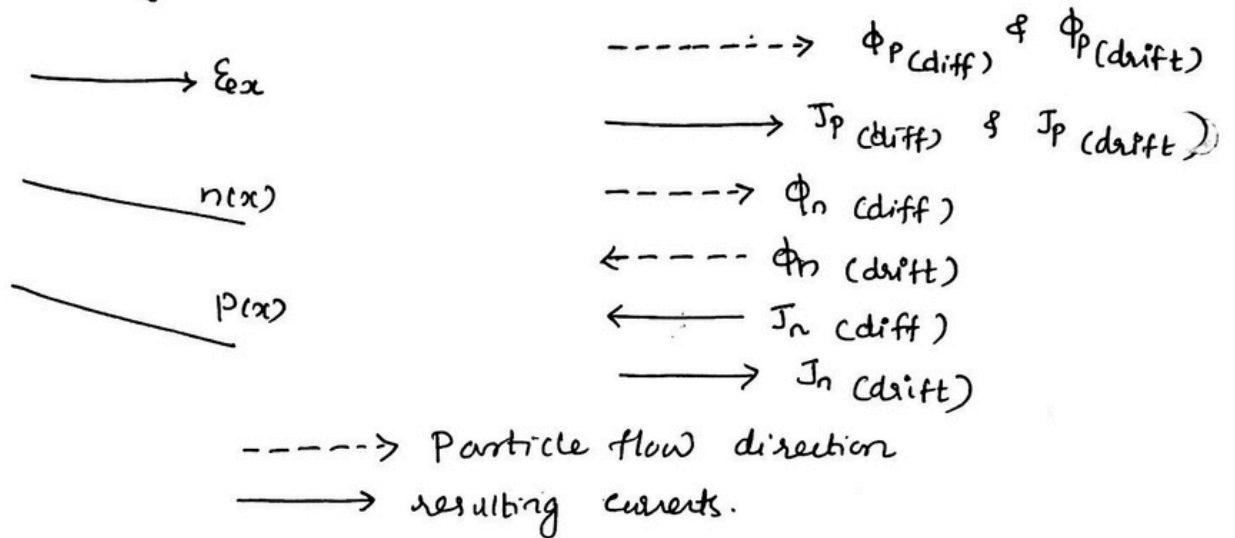
If both electric field and gradient are present, need to consider both drift and diffusion currents.

$$J_n(x) = J_n(\text{diffusion}) + J_n(\text{drift})$$

$$= q D_n \cdot \frac{d n(x)}{dx} + n q \mu_n E_x$$

$$J_p(x) = -q D_p \cdot \frac{d p(x)}{dx} + p q \mu_p E_x$$

Drift and diffusion directions for e^- s and holes in a carrier gradient and electric field.



Note:- Drift and diffusion currents are additive for holes when electric field is in the direction of decreasing concentration and is subtractive for e^- s.

Einstein relations

At equilibrium, no net current flows in a semiconductor.

ie, Diffusion current + drift current = 0

$$J_p(\text{diffusion}) + J_p(\text{drift}) = 0$$

$$-q D_p \cdot \frac{dP(x)}{dx} + P q \mu_p E(x) = 0$$

$$q D_p \cdot \frac{dP(x)}{dx} = P q \mu_p E(x)$$

$$D_p \cdot \frac{dP(x)}{dx} = P \mu_p E(x)$$

$$E(x) = \frac{D_p}{P \mu_p} \cdot \frac{dP(x)}{dx}$$

$$E(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{P} \cdot \frac{dP(x)}{dx} \quad \text{--- (1)}$$

Considering equilibrium,

$$P = P_0 \quad \text{ie, } P(x) = P_0(x)$$

$$\text{(1)} \Rightarrow E(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{P_0} \cdot \frac{dP_0(x)}{dx} \quad \text{--- (2)}$$

$$\text{we have, } P_0 = P_i \cdot e^{(E_i - E_F)/kT} \quad \text{--- (3)}$$

$$\text{(2) becomes } E(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{P_i \cdot e^{(E_i - E_F)/kT}} \cdot \frac{d}{dx} \left[P_i \cdot e^{(E_i - E_F)/kT} \right]$$
$$= \frac{D_p}{\mu_p} \cdot \frac{1}{P_i \cdot e^{(E_i - E_F)/kT}} \cdot P_i \cdot e^{(E_i - E_F)/kT} \cdot \frac{d(E_i - E_F)/kT}{dx}$$

$$= \frac{D_p}{\mu_p} \cdot \frac{1}{kT} \cdot \frac{d(E_i - E_F)}{dx}$$

$$E(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{kT} \cdot \frac{dE_i}{dx} - \frac{dE_F}{dx}$$

$$\frac{d(10e^{2x})}{dx} = 10e^{2x} \cdot \frac{d2x}{dx} = 10e^{2x} \cdot 2$$

$$E_F(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{kT} \frac{d}{dx} E_i(x) \quad \therefore \frac{dE_F}{dx} = 0 \quad \text{--- (4)}$$

we have, $V(x) = \frac{E(x)}{-q}$ --- (5) $E(x)$: Potential energy

$$E_F(x) = -\frac{d}{dx} V(x) \quad \text{--- (6)}$$

$$= -\frac{d}{dx} \frac{E(x)}{(-q)}$$

$$E_F(x) = \frac{1}{q} \frac{d}{dx} E(x) \quad \text{--- (7)}$$

$$\frac{d}{dx} E(x) = \frac{d}{dx} E_c = \frac{d}{dx} E_v = \frac{d}{dx} E_i$$

compare (7) and (4)

$$\frac{D_p}{\mu_p} \cdot \frac{1}{kT} = \frac{1}{q}$$

$$\frac{D_p}{\mu_p} = \frac{kT}{q} \quad \text{--- (8)}$$

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{q} = V_T \quad \text{: Volt equivalent of Temperature}$$

At 300 K,

$$\begin{aligned} \frac{D_n}{\mu_n} &= \frac{kT}{q} \\ &= \frac{8.62 \times 10^{-5} \text{ eV/K} \times 300}{1 \text{ eV}} \\ &= \underline{\underline{25 \text{ mV}}} \end{aligned}$$

Effect of Temperature and Doping on Mobility

1) Effect of temperature on Mobility

The two basic types of scattering mechanisms which affects the mobility of carriers are

- Lattice scattering
- Impurity scattering

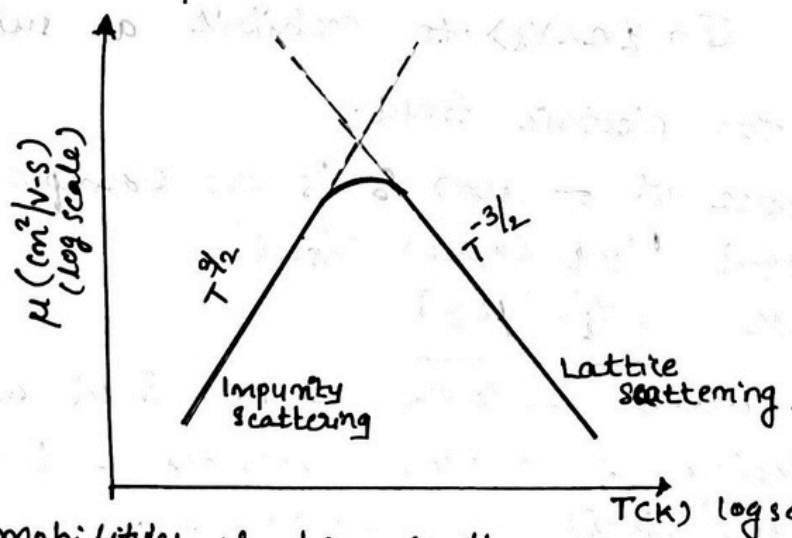
Lattice scattering:

In lattice scattering a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature. ~~So as~~ Lattice scattering increases, mobility decreases with increase in temperature. Mobility decreases with increase in temperature. i.e., $\mu \propto T^{-3/2}$

Impurity scattering:

Scattering from crystal defects, such as ionised impurities becomes the dominant mechanism at low T. since the atoms of the cooler lattice are less agitated, Lattice scattering is less important.

→ However, the thermal motion of the carriers is also slower.
 → Since a slowly moving carrier is likely to be scattered more strongly by an interaction with a charged ion than is a carrier with a greater momentum, impurity scattering cause decrease in mobility with decrease in temperature
 $\mu \propto T^{3/2}$



The mobilities of two scattering mechanisms add inversely

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

Thus the mechanism causing the lowest mobility value dominates.

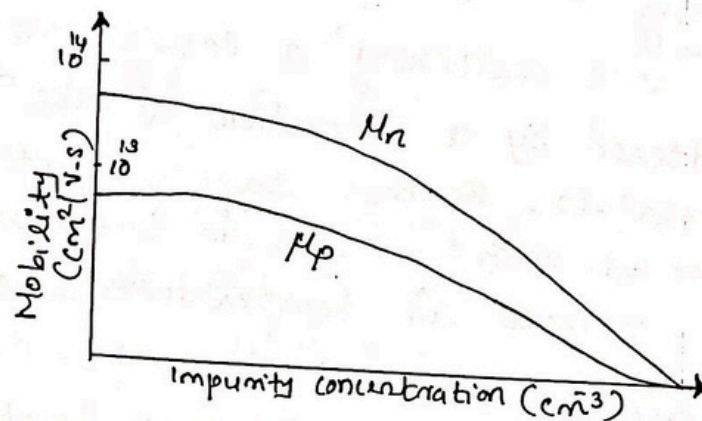
2) Effect of doping on Mobility

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As the concentration of impurities increases, the effects of impurity scattering are higher at higher temperature.

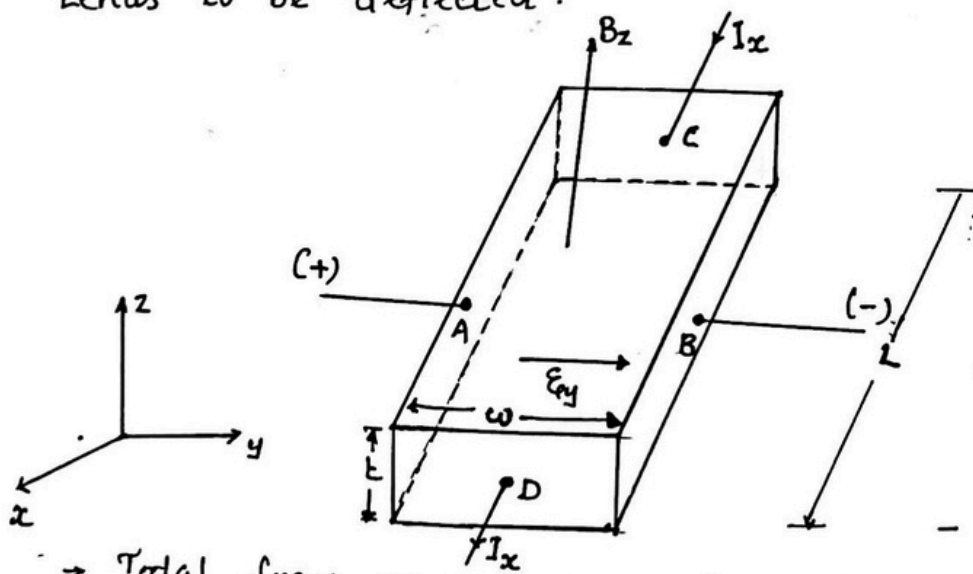
ie, μ_n at 300K is $1350 \text{ cm}^2/\text{V-s}$. With donor doping concentration, $N_D = 10^{17} \text{ cm}^{-3}$, $\mu_n = 700 \text{ cm}^2/\text{V-s}$.

→ Thus the presence of 10^{17} ionised donors introduces a significant amount of impurity scattering.



Hall Effect.

→ If a magnetic field is applied $\perp$ to the direction in which holes drift in a p-type bar, the path of the holes tends to be deflected.



→ Total force on a single hole due to electric field and magnetic field is

$$F = q(E + v \times B) \quad \text{--- (1)}$$

In the y direction, the force is

$$F_y = q(E_y - v_x B_z) \text{ — (2)}$$

$$v_x = v_y = 0 \\ B_x = B_y = 0$$

$$v \times B = \begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

From (2), it is clear that,

Unless an electric field E_y is established along the width of the bar, each hole will experience a net force (and therefore an acceleration) in the y -direction due to the $qv_x B_z$ product.

→ To maintain a steady state flow of holes down the length of the bar, the electric field E_y must just balance the product $qv_x B_z$.

$$\boxed{E_y = v_x B_z} \text{ — (3) so that } F_y = 0$$

→ Physically this electric field is set up when the magnetic field shifts the hole distribution slightly in the $-y$ direction.

→ Once the ~~max~~ electric field E_y becomes as large as $v_x B_z$, no net lateral force is experienced by the holes as they drift along the bar.

→ The establishment of the electric field E_y is known as Hall effect.

→ The resulting voltage

$$V_{AB} = E_y w \text{ — (4) is called Hall voltage.}$$

We know, $J_x = qP \langle v_x \rangle = qP_0 \langle v_x \rangle$
For holes, $\Rightarrow \langle v_x \rangle = \frac{J_x}{qP_0} \text{ — (a)}$

Substitute
(a) in (3),

$$E_y = \frac{J_x}{qP_0} B_z \text{ — (5)}$$

$$\epsilon_y = R_H I_x B_z \text{ ————— } \textcircled{6}$$

where $R_H = \frac{1}{2P_0}$ ————— (7) Hall's Coefficient

Hall field $E_y \propto I_x \cdot B_z$

└─ Magnetic flux density

└─ Current density.

From (7), $P_0 = \frac{1}{2R_H}$ — (8)

$$\textcircled{5} \Rightarrow P_0 = \frac{J_x}{2 \epsilon_0} \cdot B_z \quad \text{---} \textcircled{9}$$

$$\text{Equating, } P_0 = \frac{1}{2R_H} = \frac{J_x \cdot B_z}{2E_y} = \frac{(I_x/wt) \cdot B_z}{2(v_{AB}/w)}$$

$$P_0 = \frac{I_{nc} \cdot B_z}{2L V_{AB}} \quad \text{--- (10)}$$

$$E_y = \frac{V_{AB}}{\omega}$$

→ from (4)

$$V_{AB} = \frac{I_x B_z}{E \rho_{20}} \Rightarrow V_{AB} = \frac{I_x B_z R_H}{E} \quad \text{--- (11)}$$

$$\text{Resistivity } \rho = \frac{RA}{L} = \frac{R \cdot wt}{L} = \frac{(V_{oc}/I_{sc}) \cdot wt}{L}$$

$$P = \frac{V_{CD} I_x}{L/\omega t} \quad \text{--- (12)}$$

we have,

$$\sigma = \frac{1}{\rho} \approx 290 \text{ Hp}$$

$$\mu_p = \frac{\sigma}{q p_0} = \frac{1/p}{1/R_H} = \frac{R_H}{p}$$

$$\boxed{M_p = \frac{R_H}{P}} \quad \text{--- (13)}$$

Note :-

→ If 'n' type sc is used, a is ve.

$\rightarrow V_{AB}$ and R_H are also negative.

Applications:

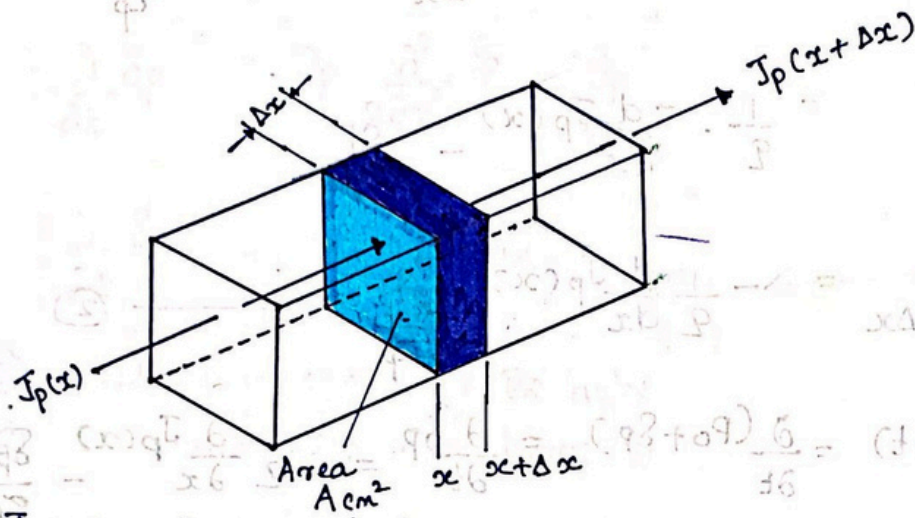
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Hall effect is used to

1. To determine the type of semiconductor material.
 2. To determine the ^{majority} carrier concentrations (p_0, n_0)
 3. to measure mobility.
-

Continuity equation

Consider a semiconductor with area of cross section A and differential length Δx .



$J_p(x)$ is the hole current density entering the elemental volume $\Delta x \cdot A$

$J_p(x + \Delta x)$ is the hole current density leaving the elemental volume $(x + \Delta x) A$.

Net change in hole concentration per unit time,

$$\text{ie, } \frac{\partial p}{\partial t} = \text{Net hole flux per unit volume entering} - \text{net hole flux per unit volume leaving} - \text{recombination rate.}$$

we have $J_p = q \phi_p$

$$\phi_p = \frac{J_p}{q} = \text{hole flux density}$$

$\therefore$ flux per volume, $\frac{\phi_p}{\Delta x} = \frac{J_p}{q \Delta x}$ which gives the number of carriers per unit volume entering the area $\Delta x \cdot A$ per unit time.

$$\text{Flux per volume leaving, } \frac{J_p(x + \Delta x)}{q \Delta x}$$

$$\left. \frac{\partial p}{\partial t} \right|_{x \rightarrow x + \Delta x} = \left(\frac{J_p(x)}{\Delta x \cdot q} - \frac{J_p(x + \Delta x)}{\Delta x \cdot q} \right) - \frac{\delta p}{\tau_p} \quad \text{--- (1)}$$

$$= \frac{1}{q} \left(\frac{J_p(x) - J_p(x + \Delta x)}{\Delta x} \right) - \frac{\delta p}{\tau_p}$$

$$= \frac{1}{q} \cdot -\frac{d}{dx} J_p(x) - \frac{\delta p}{\tau_p}$$

$$\left. \frac{\partial p}{\partial t} \right|_{x \rightarrow x + \Delta x} = -\frac{1}{q} \frac{d}{dx} J_p(x) - \frac{\delta p}{\tau_p} \quad \text{--- (2)}$$

$$\frac{\partial P(x, t)}{\partial t} = \frac{\partial (P_0 + \delta p)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p(x)}{\partial x} - \frac{\delta p}{\tau_p} \quad \text{--- (3)}$$

which is the continuity equation for holes.

ie, $\frac{\partial P(x, t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q} \frac{\partial J_p(x)}{\partial x} - \frac{\delta p}{\tau_p}$ for holes

$$\frac{\partial n(x, t)}{\partial t} = \frac{\partial \delta n}{\partial t} = +\frac{1}{q} \frac{\partial J_n(x)}{\partial x} - \frac{\delta n}{\tau_n}$$
 for es

Note: in (3),

$$\frac{\partial P(x, t)}{\partial t} = \frac{\partial (P_0 + \delta p)}{\partial t} = \frac{\partial P_0}{\partial t} + \frac{\partial \delta p}{\partial t}$$

but $\frac{\partial P_0}{\partial t} = 0$ as P_0 is constant.

$$\text{So } \frac{\partial P(x, t)}{\partial t} = \frac{\partial \delta p}{\partial t}$$

Diffusion current for holes,

$$J_p(x) = -q D_p \cdot \frac{dP(x)}{dx}$$

$$= -q D_p \cdot \frac{d(P_0 + \delta p)}{dx} = -q D_p \cdot \frac{d\delta p}{dx}$$

$$J_p(x) = -q D_p \cdot \frac{d\delta p}{dx} \quad \text{--- (4)}$$

Substitute (4) in (3),

$$\frac{\partial \delta_p}{\partial t} = -\frac{1}{2} \frac{\partial}{\partial x} \left[-2 D_p \cdot \frac{\partial \delta_p}{\partial x} \right] - \frac{\delta_p}{\tau_p}$$

$$\boxed{\frac{\partial \delta_p}{\partial t} = D_p \cdot \frac{\partial^2 \delta_p}{\partial x^2} - \frac{\delta_p}{\tau_p}} \quad \text{--- (5)}$$

$$\boxed{\frac{\partial \delta_n}{\partial t} = D_n \cdot \frac{\partial^2 \delta_n}{\partial x^2} - \frac{\delta_n}{\tau_n}} \quad \text{--- (6)}$$

(5) continuity eqn for ~~ex~~ holes

(6) continuity eqn for ~~holes~~ e^- s

Diffusion length

In steady state, (no excess carriers)

(5) and (6) becomes zero

$$\frac{\partial \delta_p}{\partial t} = 0$$

$$D_p \cdot \frac{\partial^2 \delta_p}{\partial x^2} - \frac{\delta_p}{\tau_p} = 0$$

$$\boxed{\frac{\partial^2 \delta_p}{\partial x^2} = \frac{\delta_p}{D_p \cdot \tau_p} = \frac{\delta_p}{L_p^2}} \quad \text{--- (7) Steady state diffusion eqn}$$

L_p is called diffusion length for holes.

$$L_p^2 = D_p \tau_p$$

$$\boxed{L_p = \sqrt{D_p \cdot \tau_p}} \quad \text{--- (8)}$$

From (6), $\frac{\partial^2 \delta_n}{\partial x^2} = 0$ $\frac{\partial \delta_n}{\partial t} = 0$

$$D_n \cdot \frac{\partial^2 \delta_n}{\partial x^2} - \frac{\delta_n}{\tau_n} = 0 \Rightarrow$$

$$\boxed{\frac{\partial^2 \delta_n}{\partial x^2} = \frac{\delta_n}{D_n \tau_n} = \frac{\delta_n}{L_n^2}} \quad \text{--- (9) Steady state diffusion eqn}$$

$$L_n = \sqrt{D_n \tau_n}$$

⑩

where L_n is

the diffusion length for $\bar{e}$ s.

L_p is the average distance a hole diffuses before it recombines.

L_n is the average distance an $\bar{e}$ diffuses before it recombines.

⑦ and ⑨ are called steady state diffusion eqns.

$$\textcircled{7} \Rightarrow \frac{\partial^2 \delta p}{\partial x^2} = \frac{\delta p}{L_p^2}, \text{ solution to this eqn is of}$$

the form, $\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$ — ⑪

We can evaluate C_1 and C_2 from the boundary conditions.

Excess carriers injected at $x=0$, $\delta p(x=0) = \Delta p$

$$\text{when } x=0, \delta p(x=0) = C_1 \cdot e^0 + C_2 \cdot e^0 = C_1 + C_2 = \Delta p \text{ — ⑫}$$

$$\text{when } x=\infty, \delta p = 0 = C_1 e^{\infty} + C_2 e^{-\infty}$$

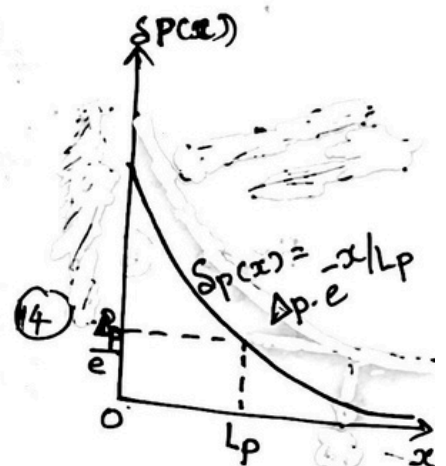
$$C_1 e^{\infty} = 0$$

$$e^{\infty} \neq 0 \text{ so } C_1 = 0 \text{ — ⑬}$$

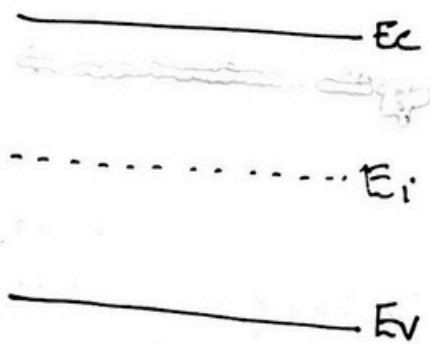
substitute ⑬ in ⑫

$$\Delta p = C_1 + C_2 = C_2$$

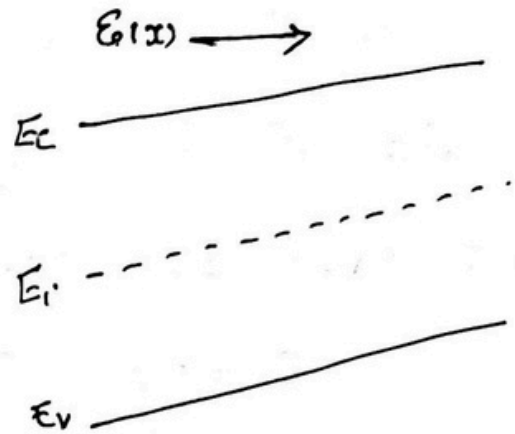
$$\text{so } \textcircled{11} \Rightarrow \delta p(x) = \Delta p \cdot e^{-x/L_p}$$



Energy band diagram of a semiconductor with Electric field



At equilibrium



under electric field $E(x)$

The applied electric field is proportional to the gradient in electron energy

Therefore energy bands bend upwards in the direction of the electric field.

Energy bands of a semiconductor will be flat when electric field in it is zero.

Poisson Equation

The Gauss' theorem in differential form is

$$\nabla \cdot \mathbf{E} = \text{div } \mathbf{E} = \rho / \epsilon_0 \quad (\text{in free space}) \text{ --- (1)}$$

In a purely Electrostatic field,

$$\mathbf{E} = -\text{grad } V = -\nabla V \text{ --- (2)}$$

↳ Electric field

ρ : Charge density
 V : Potential
 ϵ_0 : Permittivity of free space

Substitute (2) in (1)

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \nabla \cdot (-\nabla V) = \rho / \epsilon_0 \\ &= -\nabla^2 V = \rho / \epsilon_0 \end{aligned}$$

$$\boxed{\nabla^2 V = -\rho / \epsilon_0} \quad \text{Poisson eqn.}$$

In charge free region, $\rho = 0$

$$\boxed{\nabla^2 V = 0} \quad \text{Laplace eqn}$$

Q. Show that L_p is the average distance a hole diffuses before it recombines.

The probability that a hole injected at $x=0$ survives to x without recombination is the ratio of the excess carrier concentration at x and at 0. and is given by

$$\frac{\Delta p \cdot e^{-x/L_p}}{\Delta p} = e^{-x/L_p}$$

The probability that a hole recombines in a subsequent interval dx is

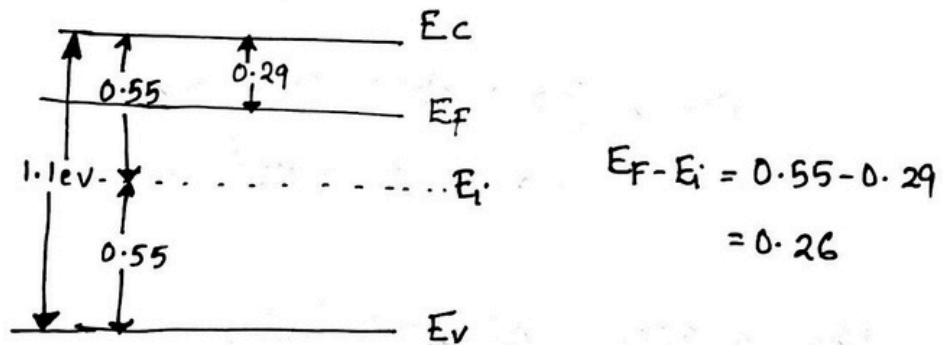
$$\begin{aligned} \frac{\delta p(x) - \delta p(x+dx)}{\delta p(x)} &= \frac{\delta p(x) - \delta p(x+dx)}{dx} \cdot \frac{dx}{\delta p(x)} \\ &= -\frac{d}{dx} \delta p(x) \cdot \frac{dx}{\delta p(x)} \\ &= -\frac{d}{dx} \Delta p \cdot e^{-x/L_p} \cdot \frac{dx}{\delta p(x)} \\ &= -\Delta p \cdot e^{-x/L_p} \cdot \left(-\frac{1}{L_p}\right) \cdot \frac{dx}{\delta p(x)} = \frac{1}{L_p} \cdot \delta p(x) \cdot \frac{dx}{\delta p(x)} \\ \delta p(x) &= \frac{1}{L_p} \cdot dx. \end{aligned}$$

The probability that a hole injected at $x=0$ recombines in a given distance dx is the product of the probability that it survives to x without recombination and the probability of recombination in subsequent interval dx .

$$\text{i.e., } P(x) = e^{-x/L_p} \cdot \frac{dx}{L_p}$$

$$\begin{aligned} \langle x \rangle &= \int_0^{\infty} x \cdot \frac{e^{-x/L_p}}{L_p} \cdot dx = \frac{1}{L_p} \left[x e^{-x/L_p} (-L_p) - \int e^{-x/L_p} (-L_p) \cdot dx \right]_0^{\infty} \\ &= \left[-x e^{-x/L_p} + e^{-x/L_p} (-L_p) \right]_0^{\infty} = \underline{\underline{L_p}}. \end{aligned}$$

1. The Fermi level position in a Si sample at 300K is 0.29 eV below E_c . Determine the carrier concentration and conductivity of the specimen. Given that $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$, $\mu_n = 1350 \text{ cm}^2/\text{V-s}$, $\mu_p = 480 \text{ cm}^2/\text{V-s}$



$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$= 1.5 \times 10^{10} \times e^{0.26/0.02586}$$

$$= \underline{\underline{3.487 \times 10^{14} \text{ cm}^{-3}}}$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{3.487 \times 10^{14}} = \underline{\underline{6.45 \times 10^5 \text{ cm}^{-3}}}$$

Conductivity, $\sigma = q (n_0 \mu_n + p_0 \mu_p)$

$$= 1.6 \times 10^{-19} [3.487 \times 10^{14} \times 1350 + 6.45 \times 10^5 \times 480]$$

$$= \underline{\underline{0.0753 \Omega^{-1} \text{ cm}^{-1}}}$$

2. A GaAs sample is doped so that the electron and hole components of currents are equal in an applied electric field. Calculate the equilibrium electron-hole concentrations, the net doping and the sample resistivity at 300K.

Given, $\mu_p = 400 \text{ cm}^2/\text{Vs}$, $\mu_n = 8500 \text{ cm}^2/\text{Vs}$
 $n_i = 1.79 \times 10^6 \text{ cm}^{-3}$

For a given electric field,

$$J_{n\text{ drift}} = J_{p\text{ drift}}$$

$$n_0 \mu_n E_x = p_0 \mu_p E_x \quad E_x \text{ is same}$$

$$\Rightarrow n_0 \mu_n = p_0 \mu_p$$

$$n_0 \mu_n = p_0 \mu_p$$

$$\mu_n = 8500 \text{ cm}^2/\text{Vs}$$

$$n_0 = p_0 \cdot \frac{\mu_p}{\mu_n}$$

$$\mu_p = 400 \text{ cm}^2/\text{Vs}$$

$$n_0 = p_0 \times \frac{400}{8500} = 0.0471 p_0 \quad \text{--- (1)}$$

We have, $n_0 p_0 = n_i^2$

$$0.0471 p_0 \times p_0 = n_i^2 = (1.79 \times 10^6)^2 = 3.2041 \times 10^{12}$$

$$0.0471 p_0^2 = 3.2041 \times 10^{12}$$

$$p_0^2 = 68 \times 10^{12} \Rightarrow p_0 = 8.25 \times 10^6 \text{ cm}^{-3}$$

$$n_0 = 0.0471 \times p_0 = 3.88 \times 10^5 \text{ cm}^{-3}$$

$p_0 > n_0$,
 Net doping, $N_A - N_D = p_0 - n_0 = 8.25 \times 10^6 - 3.88 \times 10^5 = 7.86 \times 10^6 \text{ cm}^{-3}$

Sample resistivity, $\rho = \frac{1}{\sigma} = \frac{1}{q[n_0 \mu_n + p_0 \mu_p]} = \frac{1}{1.6 \times 10^{19} [3.88 \times 10^5 \times 8500 + 8.25 \times 10^6 \times 400]} = 9.48 \times 10^8 \text{ }\Omega\text{cm}$

3. A sample of Si is doped with 10^{17} phosphorous atoms/cm³.
 What would you expect to measure for its resistivity?
 What hall voltage would you expect in a sample 100 μ m thick if $I_x = 1$ mA and $B_z = 1$ kG = 10^{-5} Wb/cm².

Given $\mu_n = 700$ cm²/V.s

$$\rho = \frac{1}{\sigma}$$

$$\begin{aligned}\sigma &= n_0 q \mu_n \\ &= 10^{17} \times 1.6 \times 10^{-19} \times 700 \\ &= 11.2 \, \Omega^{-1} \text{cm}^{-1}\end{aligned}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{11.2} = 0.0893 \, \Omega \text{cm}$$

Hall Coefficient, $R_H = \frac{1}{-q n_0}$

$$\begin{aligned}&= -\frac{1}{1.6 \times 10^{-19} \times 10^{17}} \\ &= -62.5 \, \text{cm}^3/\text{C}\end{aligned}$$

Hall voltage, $V_{AB} = \frac{I_x \cdot B_z \cdot R_H}{t}$

$$\begin{aligned}&= \frac{10^{-3} \times 10^{-5} \times -62.5}{10^{-2}} \\ &= -62.5 \, \mu\text{V}\end{aligned}$$

$$\mu_n = 700 \, \text{cm}^2/\text{V.s}$$

$$B_z = 1 \, \text{kG} = 10^{-5} \, \text{Wb/cm}^2$$

$$I_x = 1 \, \text{mA} = 10^{-3} \, \text{A}$$

$$N_D = n_0 = 10^{17} \, \text{cm}^{-3}$$

$$\begin{aligned}t &= 100 \, \mu\text{m} = 100 \times 10^{-6} \, \text{m} \\ &= 100 \times 10^{-6} \times 10^2 \, \text{cm} \\ &= 10^{-2} \, \text{cm}\end{aligned}$$

$$\rho = ?$$

$$V_{AB} = ?$$

4. Given $B_z = 6 \text{ kG}$, $I_x = 1 \text{ mA}$, $\omega = 0.2 \text{ mm}$, $t = 100 \mu\text{m}$

$L = 4 \text{ mm}$, $V_{AB} = -2 \text{ mV}$, $V_{CD} = 500 \text{ mV}$. Find the type of Semiconductor, Majority carrier concentration and mobility?

As V_{AB} is -ve, Semiconductor is n-type

$$\boxed{n_0 = \frac{I_x \cdot B_z}{q t \cdot V_{AB}}}$$

$$= 10^{-3} \times 6 \times 10^{-5}$$

$$\frac{-1.6 \times 10^{-19} \times 10^{-2} \times -2 \times 10^{-3}}{}$$

$$= 1.875 \times 10^{16} \text{ cm}^{-3}$$

$$\boxed{\rho = \frac{V_{CD}/I_x}{L/\omega t} = \frac{500 \times 10^{-3}/10^{-3}}{0.4/0.02 \times 10^{-2}}}$$

$$= 0.25 \Omega \text{ cm}$$

$$\boxed{\sigma = n_0 q \mu_n} \Rightarrow \boxed{\mu_n = \frac{\sigma}{n_0 q} = \frac{1}{\rho n_0 q}}$$

$$\mu_n \Rightarrow \frac{1}{0.25 \times 1.875 \times 10^{16} \times 1.6 \times 10^{-19}}$$

$$\mu_n = 1333.5 \text{ m}^2/\text{V-s}$$

$$I_x = 1 \text{ mA} = 10^{-3} \text{ A}$$

$$B_z = 6 \text{ kG} = 6 \times 10^5 \text{ Wb/cm}^2$$

$$t = 100 \mu\text{m} = 10^{-2} \text{ cm}$$

$$V_{AB} = -2 \text{ mV} \\ = -2 \times 10^{-3} \text{ V}$$

$$V_{CD} = 500 \text{ mV}$$

$$L = 4 \text{ mm}$$

$$= 4 \times 10^{-3} \text{ m}$$

$$= 4 \times 10^{-3} \times 10^2 \text{ cm}$$

$$= 0.4$$

$$\omega = 0.2 \text{ mm}$$

$$= 0.2 \times 10^{-3} \times 10^2 \text{ cm}$$

$$= 0.02 \text{ cm}$$

5. Electron mobility and life time in a semiconductor at room temperature are $0.36 \text{ m}^2/\text{V-s}$ and $340 \mu\text{s}$ respectively. Calculate electron diffusion length.

$$L_n = \sqrt{D_n \cdot \tau_n}$$
$$= \sqrt{D_n \times 340 \times 10^{-6}}$$

$$\mu_n = 0.36 \text{ m}^2/\text{V-s}$$

$$\tau_n = 340 \mu\text{s}$$

We have, $\frac{D_n}{\mu_n} = \frac{kT}{q}$

$$\frac{D_n}{\mu_n} = \frac{8.62 \times 10^{-5} \times 300}{1} = 0.02586$$

$$D_n = \mu_n \times 0.02586$$

$$= 0.36 \times 0.02586$$

$$= 9.31 \times 10^{-3}$$

$$L_n = \sqrt{D_n \tau_n} = \sqrt{9.31 \times 10^{-3} \times 340 \times 10^{-6}}$$

$$= 1.78 \times 10^{-3} \text{ m}$$

$$= \underline{\underline{1.78 \times 10^{-3} \text{ mm}}}$$

6. The Fermi level position in an n-type Ge film is 0.2 eV above the intrinsic Fermi level. The thickness of the film is 0.5 μm . Calculate the sheet resistance of the film. Assume $n_i = 2.5 \times 10^{13} \text{ cm}^{-3}$, $\mu_n = 3500 \text{ cm}^2/\text{Vs}$, $\mu_p = 1500 \text{ cm}^2/\text{Vs}$

$$\frac{kT}{q} = 0.025 \text{ V}$$

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$= 10^{13} \times e^{\frac{0.2}{0.025}}$$

$$= \underline{\underline{2.19 \times 10^{16} \text{ cm}^{-3}}}$$

$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(2.5 \times 10^{13})^2}{2.19 \times 10^{16}} = \underline{\underline{4.56 \times 10^9 \text{ cm}^{-3}}}$$

$$\sigma = n_0 q \mu_n + p_0 q \mu_p$$

$$= q [n_0 \mu_n + p_0 \mu_p]$$

$$= 1.6 \times 10^{-19} [2.19 \times 10^{16} \times 3500 + 4.56 \times 10^9 \times 1500]$$

$$= \underline{\underline{12.264 \times 10^1 \text{ cm}^{-1}}}$$

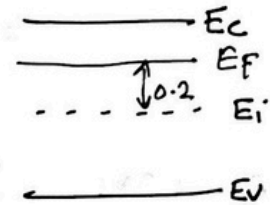
Sheet resistance is the resistance of a square ($L=W$) of thickness t

$$R = \rho \cdot \frac{L}{A} = \rho \cdot \frac{L}{W \cdot t}$$

$$\text{When } L=W, \quad \boxed{R = R_s = \frac{\rho}{t} = \frac{1}{\sigma t}}$$

$$= \frac{1}{12.264 \times 0.5 \times 10^{-4}}$$

$$= \underline{\underline{1630.78 \Omega}}$$



7. A Si bar 0.1 cm long and $100 \mu\text{m}^2$ in cross sectional area is doped with 10^{17}cm^{-3} phosphorus. Find the current at 300K with 10V applied. How long does it take an average electron to drift $1 \mu\text{m}$ in pure Si at an electric field of 100V/cm ? $\mu_n = 700 \text{cm}^2/\text{Vs}$

$$\sigma = n_0 q \mu_n = 10^{17} \times 1.6 \times 10^{-19} \times 700 = 11.2 \times 10^{-2} \text{cm}^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{11.2} = 0.0893 \Omega \text{cm}$$

$$R = \frac{\rho L}{A} = \frac{0.0893 \times 0.1}{100 \times 10^{-8}} = 8.93 \times 10^3 \Omega$$

$$I = \frac{V}{R} = \frac{10}{8.93 \times 10^3} = 1.12 \text{mA}$$

$$1 \mu\text{m}^2 = 10^{-8} \text{cm}^2$$

$$100 \mu\text{m}^2 \rightarrow 100 \times 10^{-8} \times 10^{-2} \text{cm}^2$$

$$100 \times 10^{-8}$$

$$\mu_n = 700 \text{cm}^2/\text{Vs}$$

$$E = 100 \text{V/cm}$$

$$v_d = \mu_n E = 700 \times 100 = 7 \times 10^4 \text{cm/s}$$

Low field, $t_d = \frac{L}{v_d} \Rightarrow t_d = \frac{1 \times 10^{-6} \times 10^2}{7 \times 10^4} = 1.43 \times 10^{-9} \text{s}$ $v_d = \frac{L}{t_d}$

High field, $t_d = \frac{L}{v_s} = \frac{1 \times 10^{-6} \times 10^2}{10^{17}} = 1 \times 10^{-21} \text{s}$

High field $v_d = v_s = 10^7$ (saturation velocity)

8 What is the average time taken by an electron to drift $1 \mu\text{m}$ in pure Si at an electric field of 100V/cm .

$$\mu_n = 1350 \text{cm}^2/\text{Vs}, \mu_p = 480 \text{cm}^2/\text{Vs}$$

$$v_d = \mu_n E = 1350 \times 100 = 1.35 \times 10^5 \text{cm/s}$$

Low field, $t = \frac{L}{v_d} = \frac{10^{-4}}{1.35 \times 10^5} = 7.41 \times 10^{-10} \text{s}$

High field, $t = \frac{L}{v_s} = \frac{10^{-4}}{10^{17}} = 10^{-21} \text{s}$

PN JUNCTIONS

A p-type and n type semiconductors are brought in to contact with one another to form a p-n junction.

Contact potential, V_0

Consider p and n type semiconductors, brought together to form a p-n junction. The n-material has large concentration of electrons and few holes, whereas p material has large concentration of holes and few electrons.

→ When these 2 regions are joined, diffusion of carriers takes place due to large concentration gradients at the junction.

→ Thus holes diffuse from p to n side and electrons diffuse from n to p.

→ This causes diffusion current from p to n.

→ When electrons diffuse from n to p, it leaves behind immobile or uncompensated donor ions (N_d^+) in the material.

→ Similarly, holes leaves behind uncompensated acceptor ions in the p side (N_a^-)

→ Thus a +ve charge is developed near the n side of the junction and a -ve charge near the p side.

→ This develops an electric field from +ve to -ve charge (ie, from n side to p side)

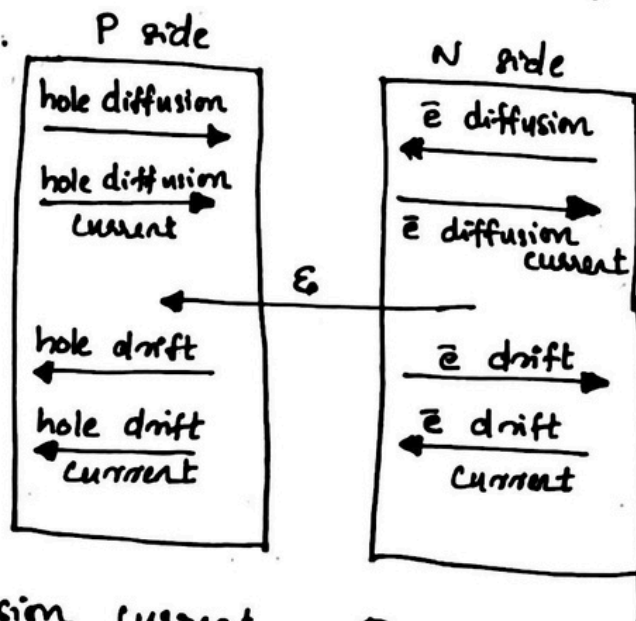
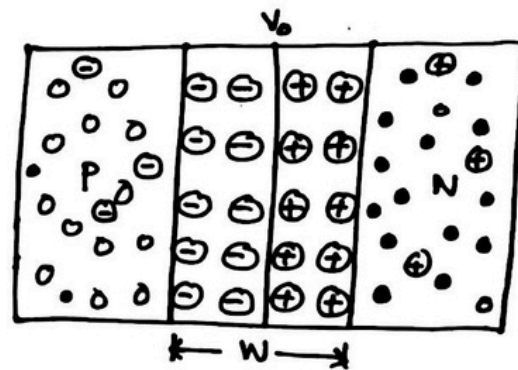
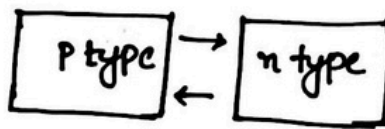
→ Due to electric field, drift current is set up from n to p.

→ At equilibrium, there is no net current flow at the junction which means the current due to drift cancels diffusion current.

$$J_{p(\text{drift})} + J_{p(\text{diffusion})} = 0$$

$$J_{n(\text{drift})} + J_{n(\text{diffusion})} = 0$$

- This electric field builds up until drift cancels diffusion.
- This is developed in a small region 'w' about the junction and it develops a potential difference 'V₀' across w.
- This V₀ is called contact potential, barrier potential, built in potential or diffusion potential.
- This region is called depletion region or transition region or space charge region.



Diffusion current

Drift current



At equilibrium,

drift current + diffusion current = 0

$$J_{p \text{ drift}} + J_{p \text{ diffusion}} = 0$$

$$p q \mu_p E(x) + -q D_p \frac{d p(x)}{dx} = 0$$

$$p q \mu_p E(x) = q D_p \frac{d p(x)}{dx}$$

$$E(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{p} \cdot \frac{d p(x)}{dx}$$

$$E(x) = \frac{kT}{q} \cdot \frac{1}{p} \frac{d p(x)}{dx} \quad \text{--- (1) (From Einstein relation, } \frac{D_p}{\mu_p} = \frac{kT}{q} \text{)}$$

$$\frac{q}{kT} E(x) = \frac{1}{p} \frac{d p(x)}{dx} \quad \text{--- (2)}$$

we have, $E(x) = -\frac{d v(x)}{dx}$ --- (3)

Substitute (3) in (2),

$$\frac{q}{kT} \cdot \left(-\frac{d v(x)}{dx}\right) = \frac{1}{p} \frac{d p(x)}{dx} \quad \text{--- (4)}$$

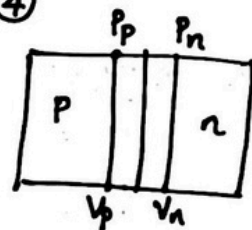
Integrating (4)

$$\frac{-q}{kT} \int_{v_p}^{v_n} \frac{d v(x)}{dx} \cdot dx = \int_{p_p}^{p_n} \frac{1}{p} \frac{d p(x)}{dx} \cdot dx$$

$$\frac{-q}{kT} [v(x)]_{v_p}^{v_n} = [\ln p]_{p_p}^{p_n}$$

$$\frac{-q}{kT} [v_n - v_p] = \ln p_n - \ln p_p$$

$$\frac{-q}{kT} (V_n - V_p) = \ln\left(\frac{p_n}{p_p}\right)$$



$$\int \frac{1}{x} dx = \ln x$$

$$\frac{q}{kT} (V_n - V_p) = \ln \left(\frac{P_p}{P_n} \right)$$

$$V_n - V_p = V_0$$

$$\frac{q}{kT} V_0 = \ln \left(\frac{P_p}{P_n} \right)$$

$$V_0 = \frac{kT}{q} \ln \frac{P_p}{P_n} \quad \text{--- (5)}$$

For ~~es~~ ~~side~~

$$V_0 = \frac{kT}{q} \ln \frac{n_n}{n_p} \quad \text{--- (6)}$$

From (5), $\frac{P_p}{P_n} = e^{qV_0/kT}$

from (6), $\frac{n_n}{n_p} = e^{qV_0/kT}$

P_p : holes on P side = N_a

P_n : holes on n side = n_i^2/N_d

n_n : es on n side = N_d

n_p : es on P side = n_i^2/N_a

In P type:

$P_p = N_a$: Majority carrier concn.

minority carrier $\bar{e} = n_p$

$$n_0 p_0 = n_i^2 \Rightarrow P_p \cdot n_p = n_i^2 \Rightarrow n_p = \frac{n_i^2}{P_p} = \frac{n_i^2}{N_a}$$

In n type:

$n_n = N_d$: Majority carrier. $\bar{e}$

P_n : minority holes.

$$n_0 p_0 = n_i^2 \Rightarrow n_n \cdot P_n = n_i^2 \Rightarrow P_n = \frac{n_i^2}{N_d}$$

$$(5) \Rightarrow V_0 = \frac{kT}{q} \ln \frac{N_a}{n_i^2/N_d}$$

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

$$(6) \Rightarrow V_0 = \frac{kT}{q} \ln \frac{N_d}{n_i^2/N_a}$$

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

Space charge at a junction

→ Maximum electric field, E_0

→ width of depletion region, W

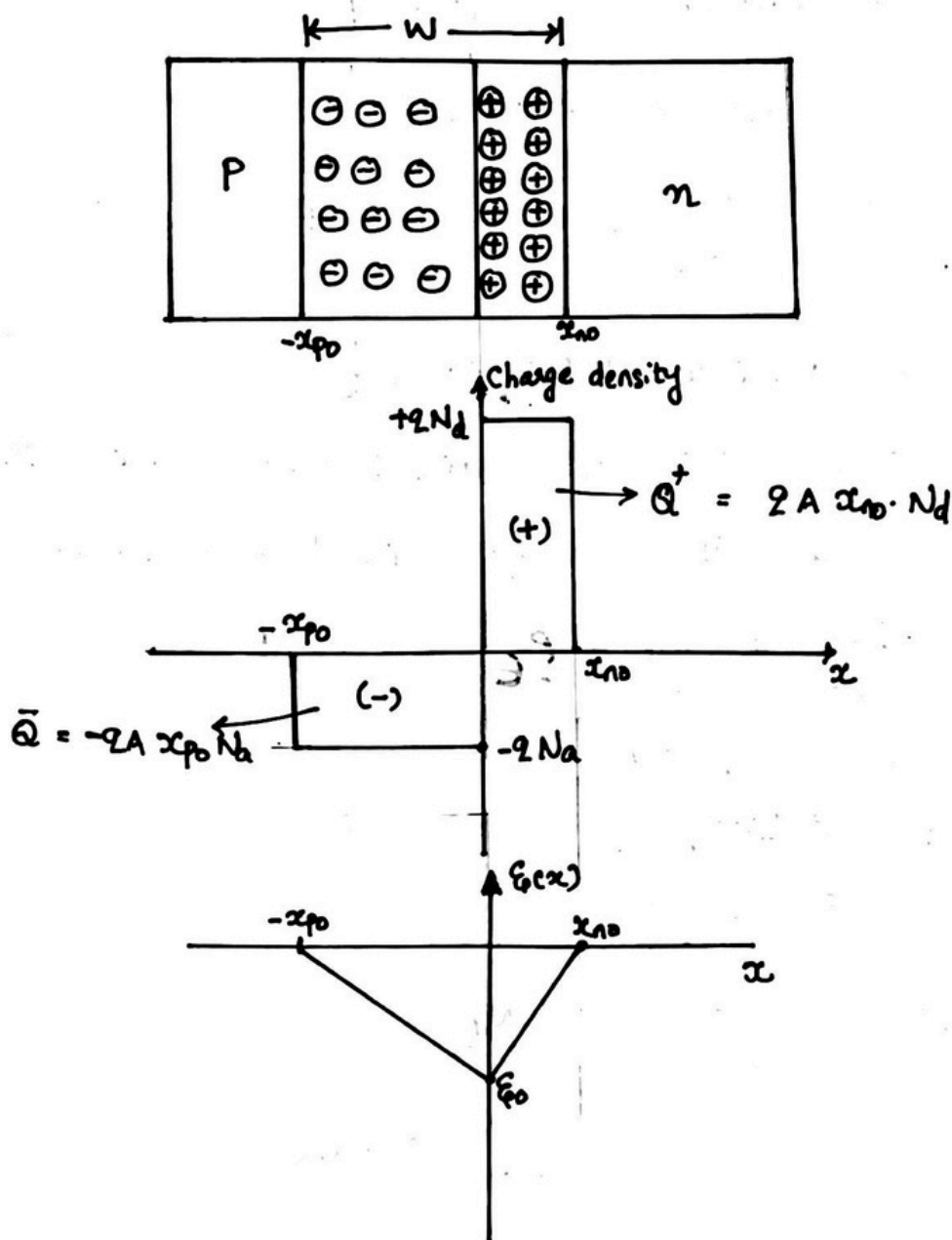
Consider a p-n junction where p is lightly doped

and n is heavily doped ($N_A < N_D$)

Consider that space charge within the transition region is only due to uncompensated donors & acceptor ions.

steady

$$N_D > N_A$$



$$\text{charge density on N side} = +q N_d$$

$$\text{charge density on P side} = -q N_a$$

$$\text{charge} = \text{charge density} \times \text{volume}$$

$$\text{charge density} = \frac{\text{charge}}{\text{volume}}$$

$$\text{charge on n side, } Q^+ = +q N_d \cdot A \cdot x_{no} \quad \text{--- (1)}$$

$$\text{charge on p side, } Q^- = -q N_a \cdot A \cdot (-x_{po})$$

$$\begin{aligned} \text{volume} &= \\ \text{n side} &\rightarrow A \cdot x_{no} \\ \text{p side} &\rightarrow -A \cdot x_{po} \end{aligned}$$

$$= q N_a \cdot A \cdot x_{po} \quad \text{--- (2)}$$

$$Q^+ = Q^-$$

$$+q N_d \cdot A \cdot x_{no} = -q N_a \cdot A \cdot x_{po}$$

$$N_d \cdot x_{no} = N_a \cdot x_{po} \quad \text{--- (3)}$$

$$\boxed{\frac{x_{no}}{x_{po}} = \frac{N_a}{N_d}} \quad \text{--- (4)}$$

Equations for Maximum electric field, E_0

The electric field distribution within the transition region is given by Poisson's equation.

$$-\frac{d^2V}{dx^2} = \frac{dE}{dx} = \frac{\rho}{\epsilon} \quad \text{--- (5)}$$

ρ - charge density

ϵ - Permittivity of the medium

$$(5) \Rightarrow \frac{dE}{dx} = \frac{\rho}{\epsilon}$$

$$\text{On n side, } \frac{dE}{dx} = \frac{+q N_d}{\epsilon} \quad \text{--- (6)}$$

$$\int_{E_0}^0 \frac{dE}{dx} \cdot dx = \frac{q}{\epsilon} \int_0^{x_{no}} N_d \cdot dx$$

$$[0 - \epsilon_0] = \frac{q}{\epsilon} \cdot N_d \cdot (x_{n0} - 0)$$

$$-\epsilon_0 = \frac{q}{\epsilon} N_d \cdot x_{n0}$$

$$\epsilon_0 = -\frac{q}{\epsilon} N_d \cdot x_{n0} \quad \text{--- (7)}$$

On p side,

$$\frac{d\epsilon}{dx} = -\frac{q N_a}{\epsilon}$$

$$\int_0^{\epsilon_0} \frac{d\epsilon}{dx} \cdot dx = \int_{-x_{p0}}^0 -\frac{q N_a}{\epsilon} \cdot dx$$

$$(\epsilon_0 - 0) = -\frac{q}{\epsilon} N_a \cdot (0 - -x_{p0})$$

$$\epsilon_0 = -\frac{q}{\epsilon} N_a \cdot x_{p0} \quad \text{--- (8)}$$

From (3), $N_d \cdot x_{n0} = N_a \cdot x_{p0}$

So (7) = (8),

$$\boxed{\epsilon_0 = -\frac{q}{\epsilon} N_d \cdot x_{n0} = -\frac{q}{\epsilon} N_a \cdot x_{p0}}$$

To find V_0 ,

$$\epsilon(x) = -\frac{dV(x)}{dx}$$

$$\int_{-x_{p0}}^{x_{n0}} \epsilon(x) dx = -V_0 = \text{Area.}$$

Area under $\epsilon(x)$ vs x is $\text{Area} = -V_0 = \frac{1}{2} W \cdot \epsilon_0 \quad \because \frac{1}{2}bh$

$$V_0 = -\frac{1}{2} W \epsilon_0 = -\frac{1}{2} W \cdot \left(-\frac{q}{\epsilon} N_d \cdot x_{n0}\right)$$

$$V_0 = \frac{1}{2} \frac{W \cdot q N_d \cdot x_{n0}}{\epsilon} = \frac{1}{2} \frac{W \cdot q N_a \cdot x_{p0}}{\epsilon}$$

--- (9)

To find w

$$w = x_{p0} + x_{n0}$$
$$= x_{p0} \left(1 + \frac{x_{n0}}{x_{p0}} \right)$$

$$w = x_{p0} \left(1 + \frac{N_a}{N_d} \right)$$

$$x_{p0} = \frac{w}{1 + \frac{N_a}{N_d}} = \frac{w}{\frac{N_a + N_d}{N_d}}$$

$$x_{p0} = w \cdot \frac{N_d}{N_a + N_d} \quad \text{--- (10)}$$

$$x_{n0} = w \cdot \frac{N_a}{N_a + N_d} \quad \text{--- (11)}$$

Substitute (10) in (9)

$$V_0 = \frac{1}{2} w \cdot \frac{q}{\epsilon} N_d \cdot x_{n0}$$
$$= \frac{1}{2} w \cdot \frac{q}{\epsilon} N_d \cdot w \cdot \frac{N_a}{N_d + N_a}$$

$$V_0 = \frac{1}{2} w^2 \frac{q}{\epsilon} \frac{N_a N_d}{N_a + N_d} \quad \text{--- (12)}$$

$$w^2 = 2 \frac{V_0 \epsilon}{q} \frac{N_a + N_d}{N_a N_d} \quad /$$

$$w = \left(2 \frac{V_0 \epsilon}{q} \frac{N_a + N_d}{N_a N_d} \right)^{1/2} \quad \text{--- (13)}$$

we have $V_0 = \frac{kT}{2} \ln \frac{N_a N_d}{n_i^2}$

$$(13) \Rightarrow w = \left(2 \frac{kT}{2} \ln \frac{N_a N_d}{n_i^2} \cdot \frac{\epsilon}{q} \frac{N_a + N_d}{N_a N_d} \right)^{1/2}$$

$$(10) \Rightarrow x_{p0} = w \cdot \frac{N_d}{N_a + N_d}$$

Substitute (13) in (10)

$$x_{p0} = \left(\frac{2\epsilon V_0}{q} \frac{N_a + N_d}{N_a N_d} \right)^{1/2} \cdot \frac{N_d}{N_a + N_d}$$

$$= \left(\frac{2\epsilon V_0}{q} \frac{N_a + N_d}{N_a N_d} \cdot \frac{N_d^2}{(N_a + N_d)^2} \right)^{1/2}$$

$$= \frac{2\epsilon V_0 N_d}{q}$$

$$x_{p0} = \left(\frac{2\epsilon V_0}{q} \frac{N_d}{N_a (N_a + N_d)} \right)^{1/2} \quad (14)$$

$$x_{n0} = \left(\frac{2\epsilon V_0}{q} \frac{N_a}{N_d (N_a + N_d)} \right)^{1/2} \quad (15)$$

$$W = x_{p0} + x_{n0}$$

$$= \left(\frac{2\epsilon V_0}{q} \frac{N_d}{N_a (N_a + N_d)} \right)^{1/2} + \left(\frac{2\epsilon V_0}{q} \frac{N_a}{N_d (N_a + N_d)} \right)^{1/2}$$

Energy Band diagram

1) Equilibrium.

2) Forward bias

3) Reverse bias.

Equilibrium → when no bias is applied.

Forward bias → when p-n junction is connected with a forward bias voltage ' V_f '. built in potential ' V_0 ' changes to ' $V_0 - V_f$ '. This is due to the external electric field due to biasing opposes the built in potential, thus canceling the overall electric field to some extent.

ie, $E \downarrow V_0 \downarrow W \downarrow$

Since barrier is lowered, more carriers diffuse from n to p and p to n . This diffusion current is large in FB.
 → As external field opposes internal field, total field reduces, drift current is reduced in FB.

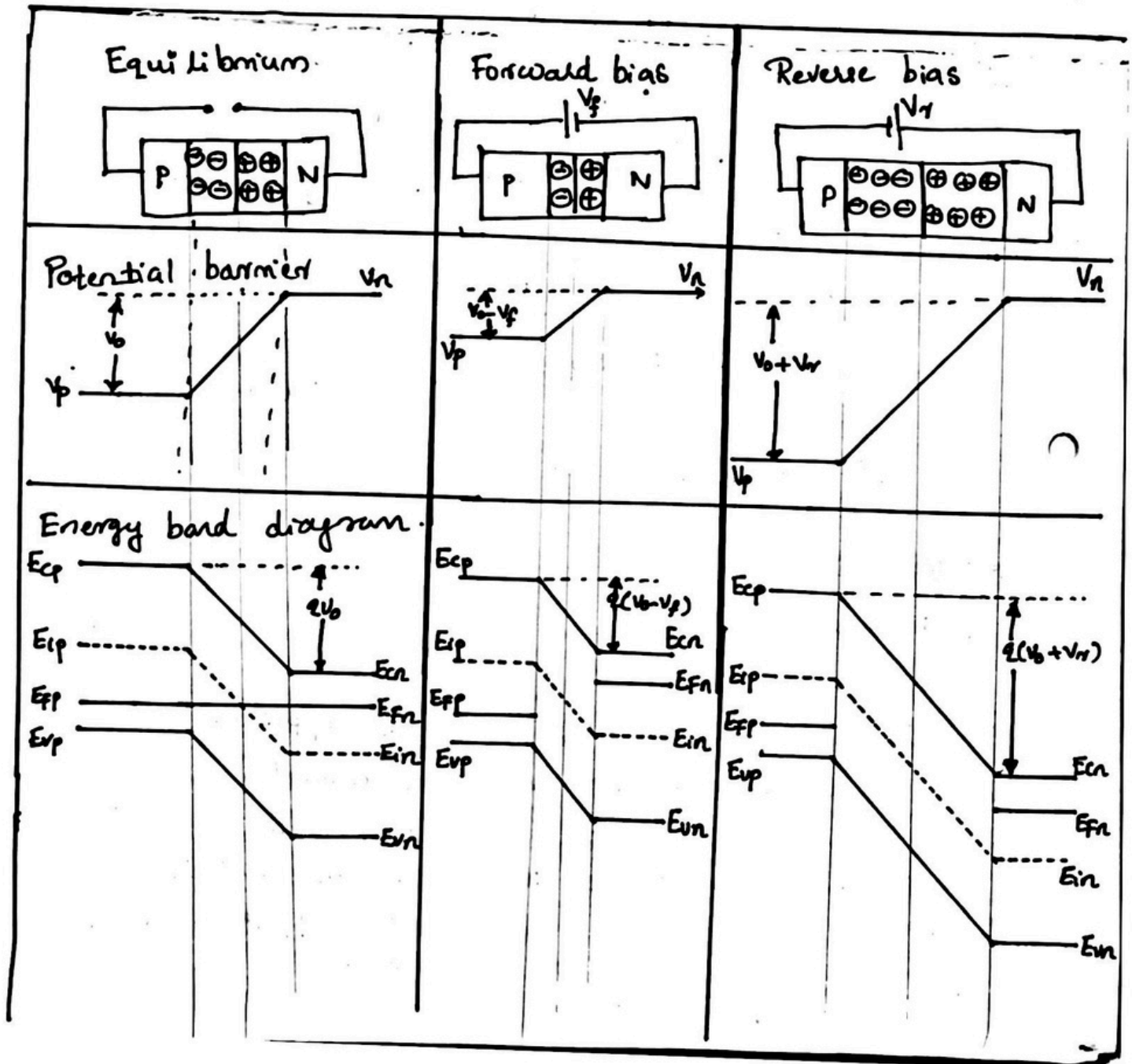
Reverse bias:

V_0 becomes $V_0 + V_R$ where V_R is the reverse bias voltage.

Since internal and external electric fields are in the same direction, $\mathcal{E} \uparrow \Rightarrow W \uparrow \Rightarrow V_0 \uparrow$

→ diffusion current is negligible in RB.

→ Drift current is more in RB



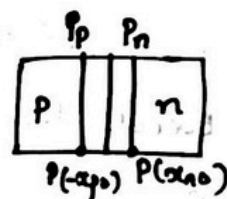
Ideal diode Equation

Approximations considered for diode equation:

1. Diode is a long diode i.e. length of $n/p \gg L_p/L_n$
2. The depletion region is completely depleted of mobile charge carriers and the region outside the depletion layer is perfectly neutral.
3. The P-n junction is abrupt (step graded)
4. The contacts at the two ends are ohmic.
5. Low level injection condition is considered for the forward bias.
6. No generation or recombination within the depletion region.
7. Diode is in steady state condition (i.e. applied voltage is dc)
8. The temperature and doping are in such a way that the impurities are completely ionized.

Derivation:

When forward bias is applied, holes from P side move in to n side. Once they reach the 'n' side, holes become minority carriers until they reach diffusion length L_p .



We have $V_0 = \frac{KT}{q} \ln \frac{P_p}{P_n}$ at equilibrium

$$\text{So } \frac{P_p}{P_n} = e^{\frac{2V_0/KT}{} } \quad \text{①}$$

When bias is applied,

$$\frac{P(-x_{p0})}{P(x_{n0})} = e^{\frac{2(V_0 - V_f)/KT}{} } \quad \text{where } V_f \text{ is the forward bias voltage.} \quad \text{②}$$

$$\text{①/②, } \frac{P_p/P_n}{P(-x_{p0})/P(x_{n0})} = \frac{e^{\frac{2V_0/KT}{} }}{e^{\frac{2(V_0 - V_f)/KT}{} }} = e^{\frac{2V_f/KT}{} } \quad \text{③}$$

For low level injection, $P_p = P(-x_{p0})$
 i.e. concentration of holes in neutral region has
 no change compared to holes at $-x_{p0}$.

$$\textcircled{3} \Rightarrow \frac{P_p/P_n}{P_p/P(x_{n0})} = e^{2V_f/kT}$$

$$\frac{P(x_{n0})}{P_n} = e^{2V_f/kT}$$

$$V_f = V$$

$$\boxed{P(x_{n0}) = P_n \cdot e^{2V/kT}} \quad \textcircled{4}$$

The above equation implies minority hole concentration at the edge of the transition region on n side ($P(x_{n0})$) is greatly increased during forward bias compared to equilibrium values (P_n)

The exponential increase in hole concentration at x_{n0} with forward bias is an example of minority carrier injection.

Excess hole concentration ΔP_n at the edge of transition region is

$$\Delta P_n = P(x_{n0}) - P_n \quad \textcircled{5}$$

$$= P_n \cdot e^{2V/kT} - P_n$$

$$\boxed{\Delta P_n = P_n (e^{2V/kT} - 1)} \quad \textcircled{6}$$

Similarly for excess e^- on p side,

$$\boxed{\Delta n_p = n_p (e^{2V/kT} - 1)} \quad \textcircled{7}$$

Distance measured in the n material in 'x' direction from $x_{n0} = x_n$

Distance in the p material in the '-x' direction from $-x_{p0}$ is $= +x_p$

we have, $\delta p(x) = \Delta P_n \cdot e^{-x/L_p}$ ——— (8)

so $\delta p(x_n) = \Delta P_n \cdot e^{-x_n/L_p}$ ——— (9)

but $\Delta P_n = P_n (e^{qV/kT} - 1)$

so (9) $\Rightarrow \delta p(x_n) = P_n (e^{qV/kT} - 1) \cdot e^{-x_n/L_p}$

(9) $\Rightarrow \delta p(x_n) = \Delta P_n \cdot e^{-x_n/L_p}$ ——— (9)

$\delta n(x_p) = \Delta n_p \cdot e^{-x_p/L_n}$ ——— (10)

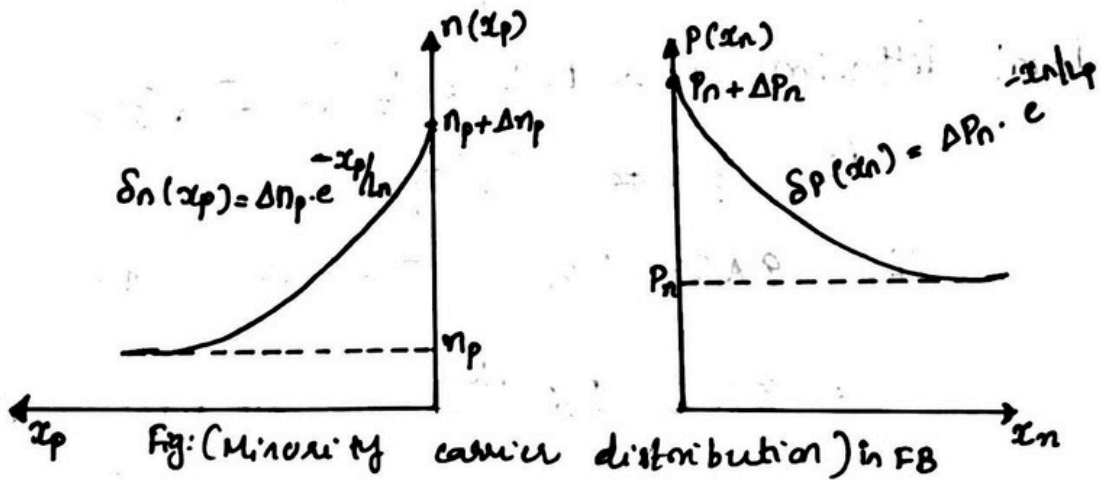
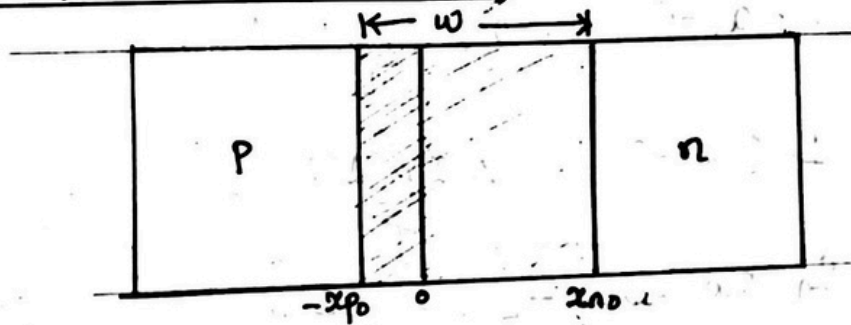


Fig: (Minority carrier distribution) in FB

To find diffusion current,

we have, $I_p(x) = -q A D_p \cdot \frac{d}{dx} P(x)$

so $I_p(x_n) = -q A \cdot D_p \cdot \frac{d}{dx} [P_0 + \delta p]$

$= -q A \cdot D_p \cdot \frac{d}{dx} \delta p(x_n)$

$= -q A D_p \cdot \frac{d}{dx} [\Delta P_n \cdot e^{-x_n/L_p}]$

$\frac{d}{dx} P_0 = 0$

$$I_p(x_n) = -2A D_p \cdot x \cdot \frac{1}{L_p} \cdot \Delta P_n \cdot e^{-x_n/L_p}$$

$$I_p(x_n) = 2A \frac{D_p}{L_p} \cdot \delta p(x_n) \quad \text{--- (11)}$$

$$\text{iii} \downarrow \quad I_n(x_p) = -2A \frac{D_n}{L_n} \cdot \delta n(x_p) \quad \text{--- (12)}$$

$$\text{(11)} \Rightarrow \text{at } x_n=0,$$

$$I_p(x_n=0) = 2A \cdot \frac{D_p}{L_p} \cdot \Delta P_n \cdot e^{-x_n/L_p}$$

$$I_p(x_n=0) = 2A \cdot \frac{D_p}{L_p} \cdot \Delta P_n \cdot e^{-0/L_p}$$

$$I_p(x_n=0) = 2A \frac{D_p}{L_p} \Delta P_n \quad \text{--- (13)}$$

$$\text{iii} \downarrow \quad I_n(x_p=0) = -2A \frac{D_n}{L_n} \Delta n_p \quad \text{--- (14)}$$

Total diffusion current, $I = I_p(x_{n0}) + I_n(x_{p0})$

ie, $I_p(x_{n0}) - I_n(x_{p0})$

$$I = 2A \frac{D_p}{L_p} \Delta P_n - 2A \frac{D_n}{L_n} \Delta n_p$$

$$= 2A \frac{D_p}{L_p} P_n (e^{qV/KT} - 1) - 2A \frac{D_n}{L_n} n_p (e^{qV/KT} - 1)$$

$$= 2A \left[\frac{D_p}{L_p} \cdot P_n + \frac{D_n}{L_n} \cdot n_p \right] (e^{qV/KT} - 1)$$

I_0

$$I = I_0 \cdot (e^{qV/KT} - 1) \quad \text{--- (15)}$$

which is the ideal diode eqn.

In reverse bias, 'V' changes to -V.

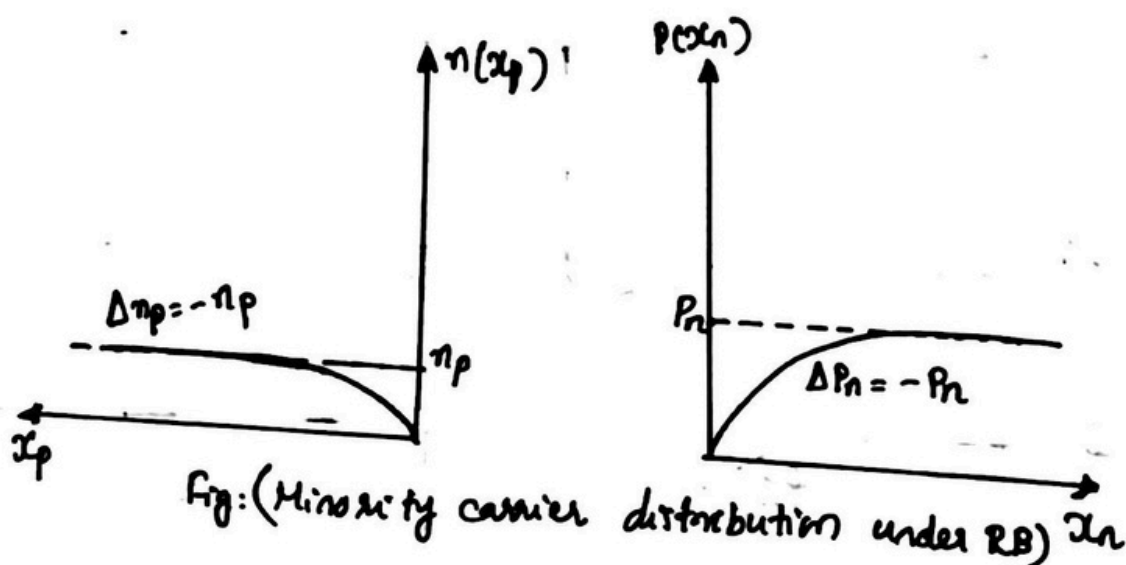
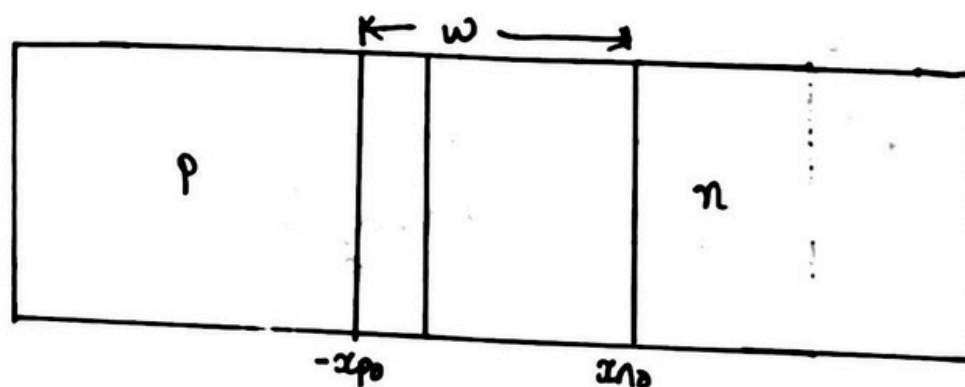
$$I = I_0 \cdot (e^{-qV/KT} - 1)$$

If V_r is much larger than kT/q , total current is the reverse saturation current.

$$I = I_0 \cdot (e^{-\text{high}} - 1) = -I_0$$

$$\textcircled{6} \Rightarrow \Delta p_n = p_n \cdot (e^{-2V_r/kT} - 1) = -p_n$$

$$\textcircled{7} \Rightarrow \Delta n_p = n_p \cdot (e^{-2V_r/kT} - 1) = -n_p$$



Metal Semiconductor Contacts:

Two types:

1. Schottky contact (Rectifying contact)
2. Ohmic contact (Non rectifying)

Work function: It is the energy required to free an e^- at Fermi level to a point in vacuum.

ϕ_m : Metal work function

ϕ_s : Semiconductor work function.

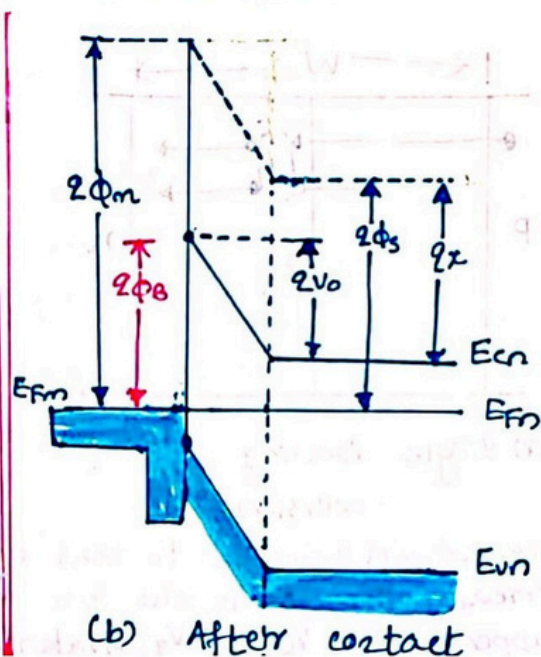
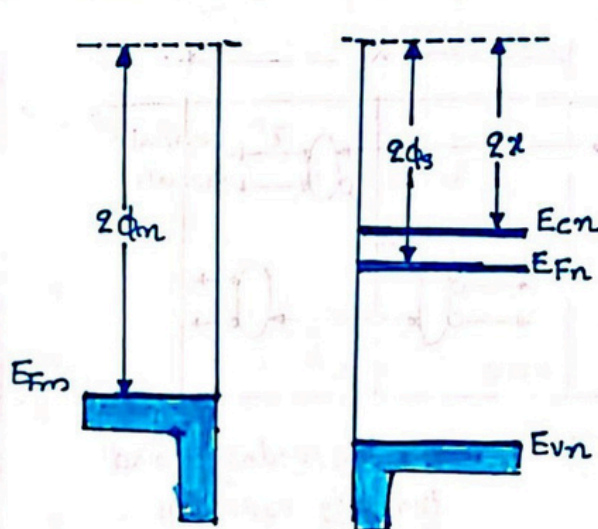
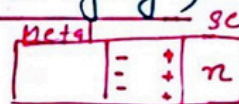
Electron affinity (χ): It is the energy required to remove an e^- at the bottom of CB to a point in vacuum.

Vacuum level is called reference level.

	Schottky contact	Ohmic contact
Metal - n type sc	$\phi_m > \phi_s$	$\phi_m < \phi_s$
Metal - p type sc	$\phi_m < \phi_s$	$\phi_m > \phi_s$

Schottky contact (Rectifying)

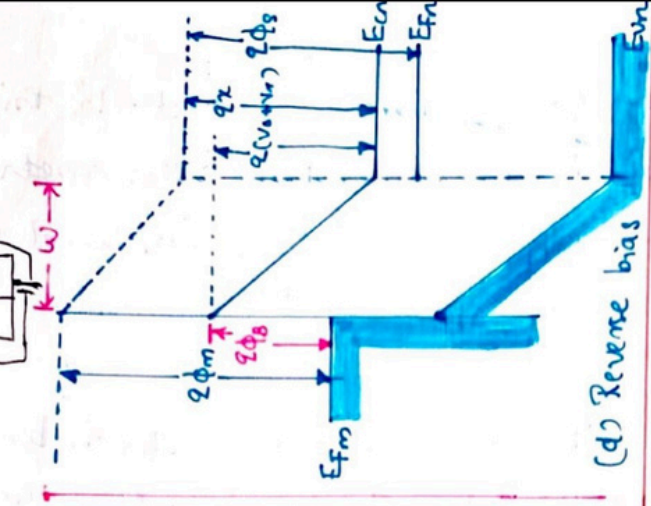
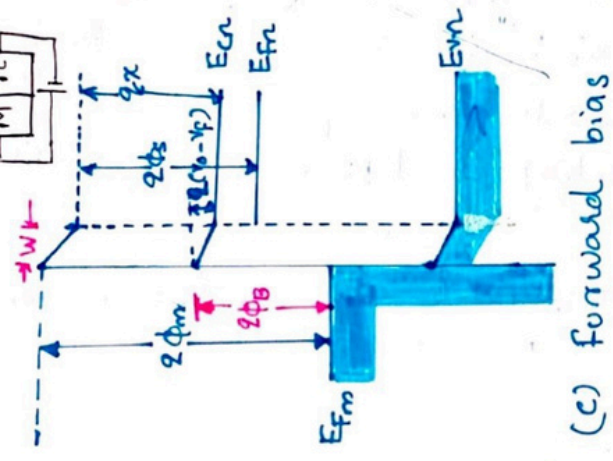
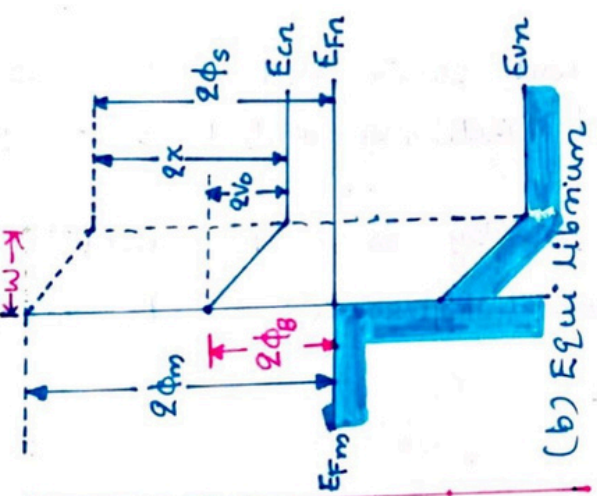
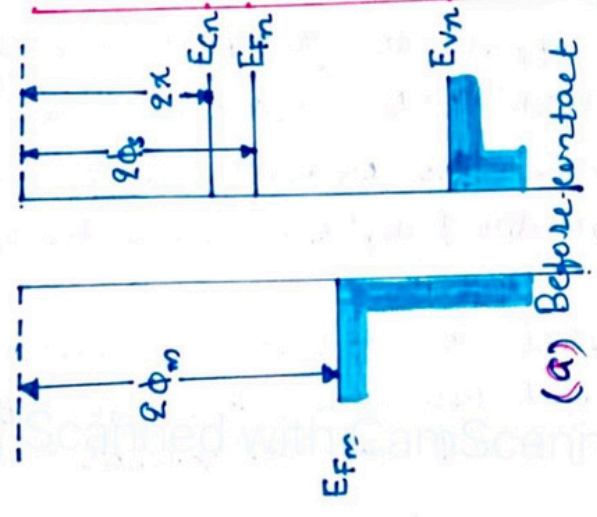
1. Metal n type sc $\phi_m > \phi_s$



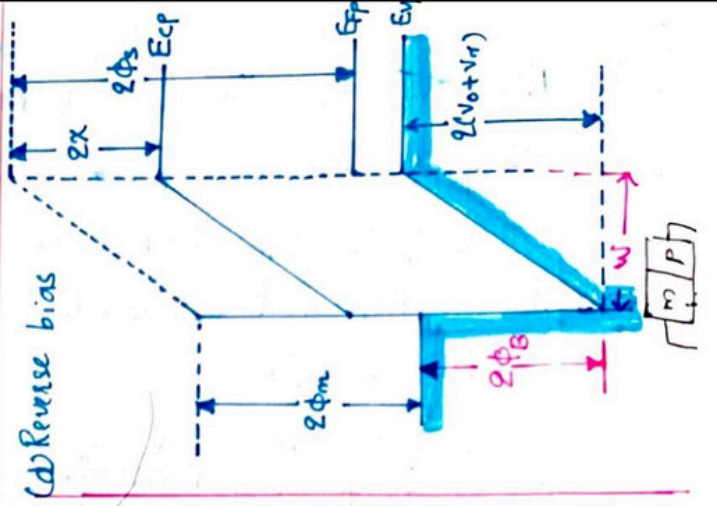
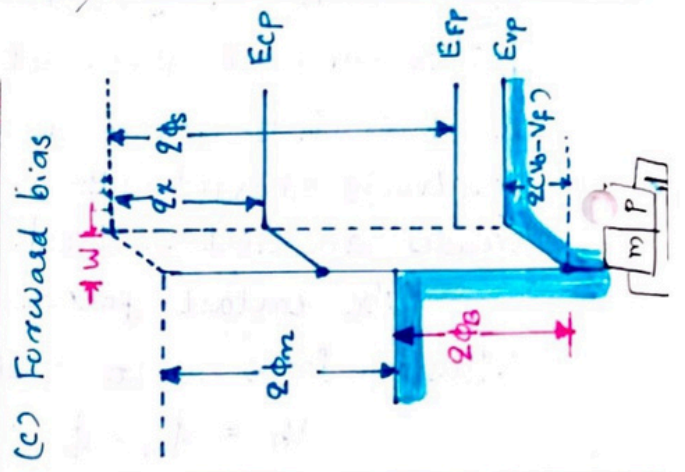
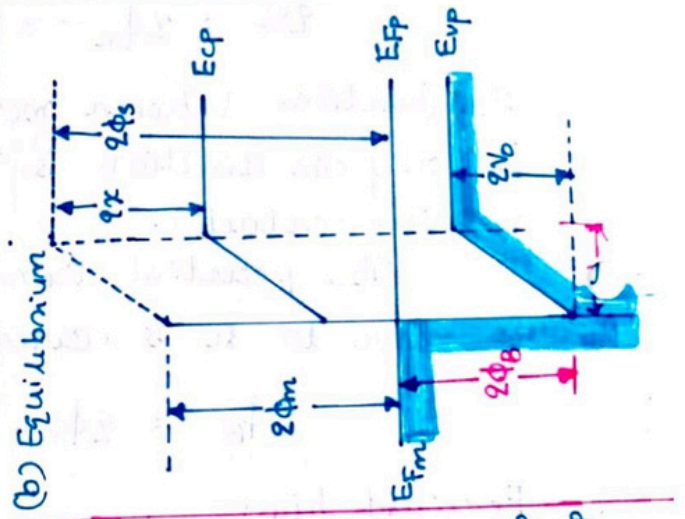
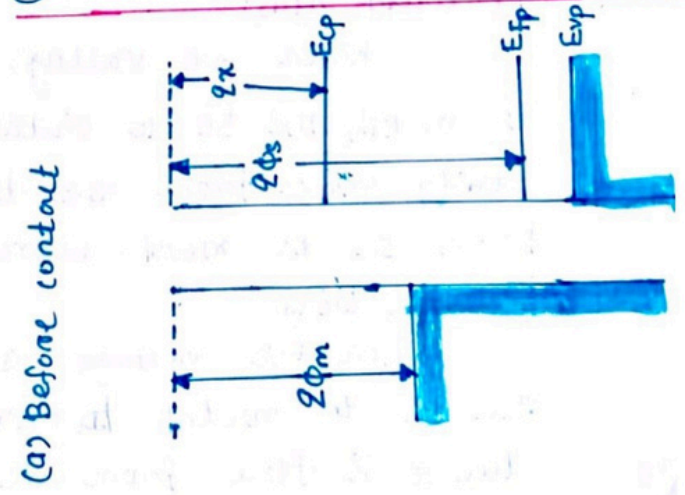
(a) Before contact

(b) After contact

Metal-n type SC : Schottky contact - $\phi_m > \phi_s$



Metal - P type SC : Schottky contact : $\phi_m < \phi_s$



Fig(a) shows metal and n -type sc before contact. In this case, Fermi level in the sc is above that in the metal ($\phi_m > \phi_s$). When both are made to contact, Fermi level in the metal and sc needs to become constant (thermal equilibrium).

So whenever a metal with work function $2\phi_m$ is brought in contact with ^{a sc with} work function $2\phi_s$, charge transfers until the Fermi level aligns at equilibrium. So the e^- s from the sc flows in to lower energy states in the metal leaving behind positively charged atoms creating a depletion region. This creates an electric field.

The contact potential V_0 prevents any further electron diffusion from sc to metal.

$$V_0 = \phi_m - \phi_s$$

$$2V_0 = 2\phi_m - 2\phi_s$$

The junction between metals and semiconductor, which conducts in only one direction is called rectifying contact (Schottky barrier contact).

The potential barrier seen by e^- s in the metal trying to move to sc is called potential barrier height ($2\phi_B$).

$$2\phi_B = 2\phi_m - \phi_x$$

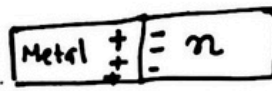
Forward bias:

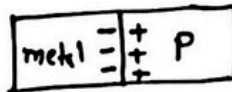
When -ve voltage is applied to n -type sc with respect to metal, the sc to metal barrier height decreases. e^- can easily move from sc to metal. The current due to e^- flow from sc to metal increases, width of depletion region decreases.

Reverse bias

When +ve voltage is applied to n -type sc w.r.t metal, the sc to metal barrier height increases. The current due to e^- flow from sc to metal decreases, width of depletion region increases.

Ohmic contact

Metal- n type : $\phi_m < \phi_s \rightarrow$ 

Metal- P type : $\phi_m > \phi_s \rightarrow$ 

- Low resistance junction provides conduction in both direction between the metal and sc.
- At thermal equilibrium, electrons from the metal move in to lower energy states of n-type sc, resulting equilibrium Fermi level.
- This raises the sc Fermi level energies relative to metals. There is no depletion region formed in this case. A small barrier is formed which can be easily overcome by applying a small external bias voltage.

Forward bias:

When -ve voltage is applied to the n type sc w.r.t metal, large no. of e^- s move from sc to the metal. Such a junction is called Ohmic junction with Forward bias.

Reverse bias:

When +ve voltage is applied to the n type sc w.r.t metal, e^- s can easily flow from the metal to the sc. Such a junction is called Ohmic junction with reverse bias.

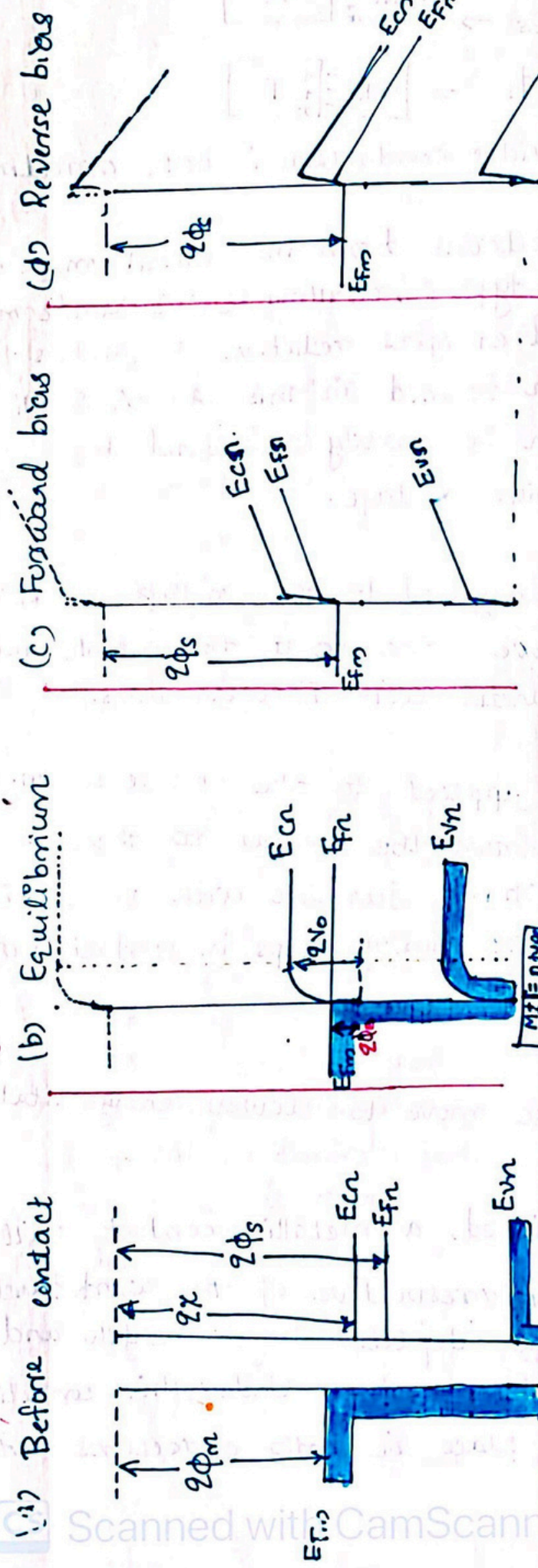
- Thus both forward and reverse biasing helps in conduction of current.

Metal P type: $\phi_m > \phi_s$

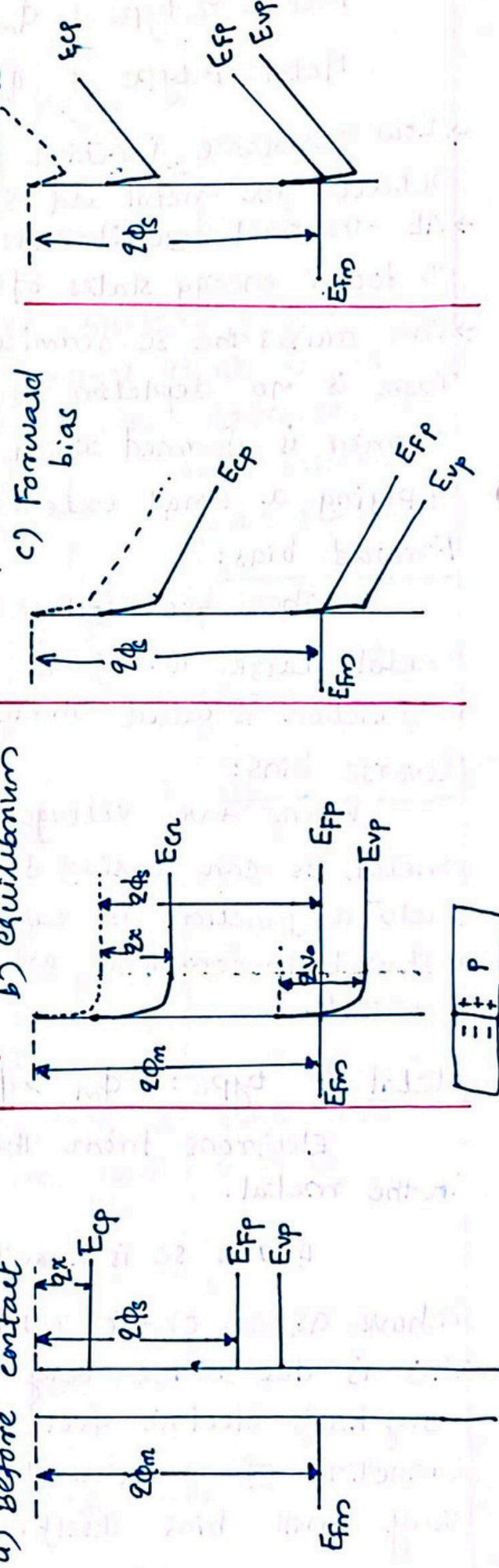
Electrons from the sc move to lower energy states in the metal.

If the sc is heavily doped, a metallic contact will behave as an Ohmic contact, irrespective of the work function. This is due to the very small depletion layer width and very high electric field at the junction. Under this condition, tunnelling of carriers takes place in both directions with very small bias itself.

Metal- n type Ohmic contact $\phi_m < \phi_s$



Metal- P type Ohmic Contact : $\phi_m > \phi_s$



No depletion region formed, bidirectional current flow

Current Voltage Characteristics

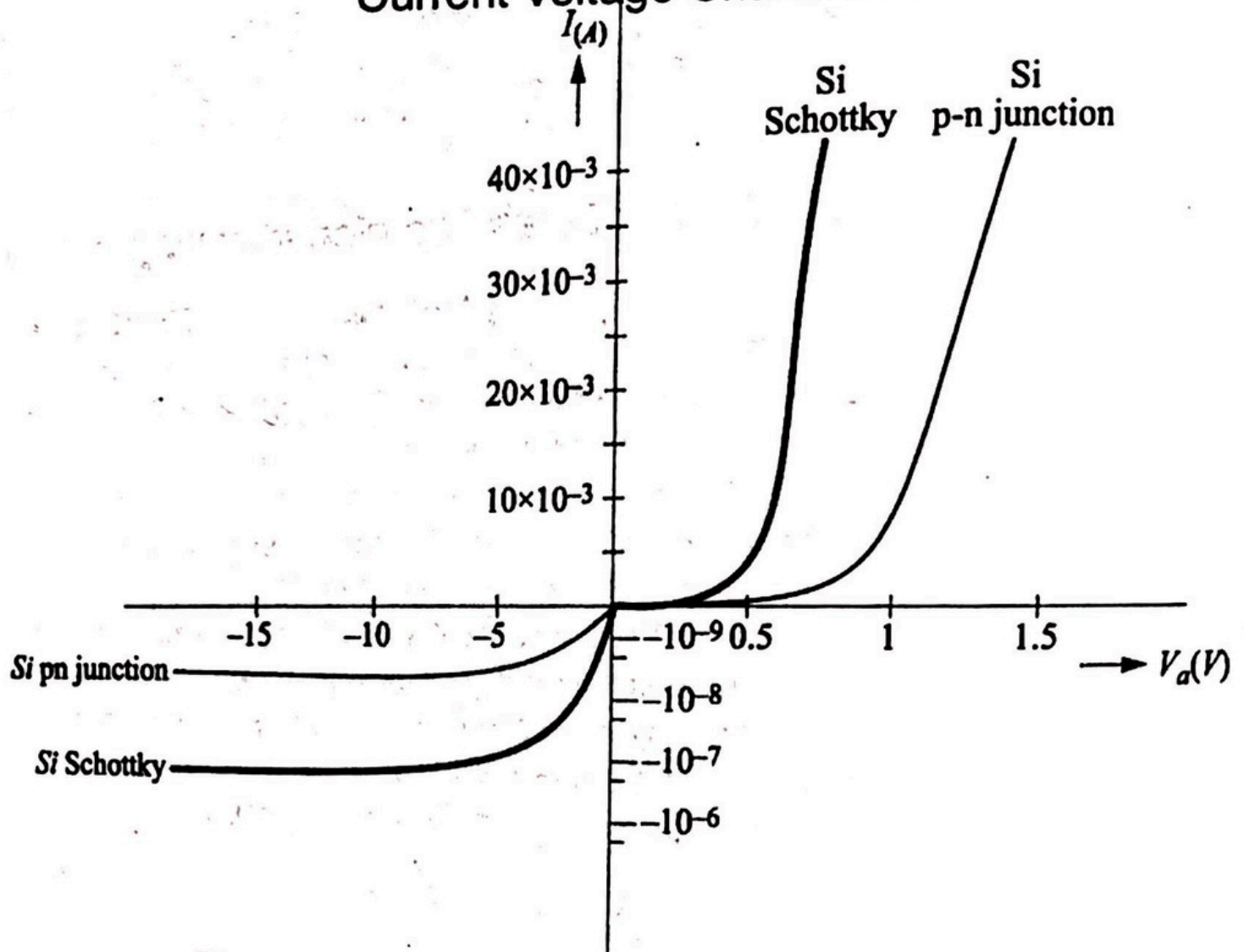


Fig. 3.29 I - V Characteristics of Si p-n junction and Si Schottky diode

Bipolar Junction Transistors (BJTs)

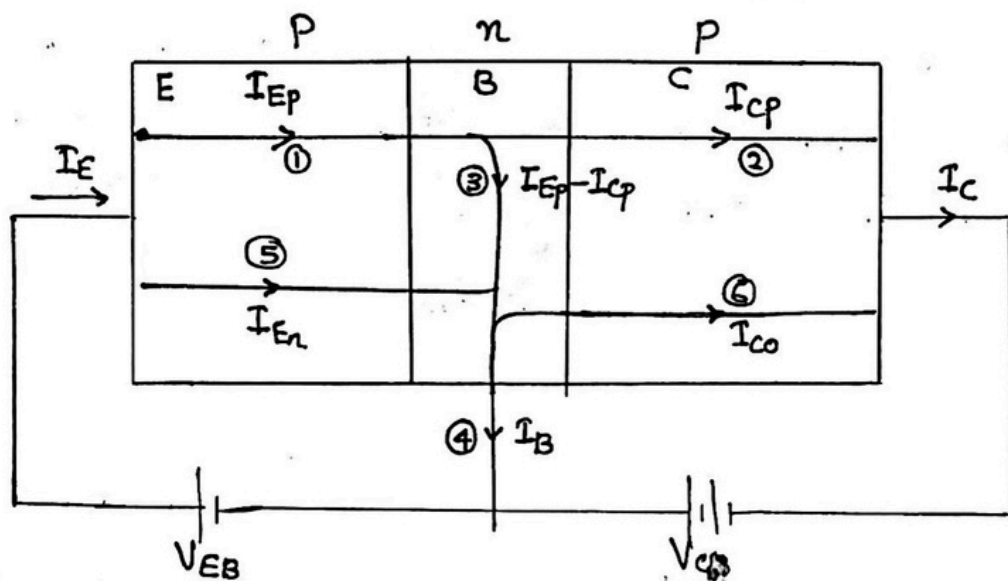
Transistors $\begin{cases} \text{BJT} \\ \text{FET} \end{cases}$

BJT has 3 separately doped regions and 2 p-n junctions. Since both e^- s and holes are involved in this device, it is called bipolar transistors.

The three terminals are called emitter, base and collector.

- The emitter has largest doping concentration
- Collector is moderately doped
- Base is lightly doped.
- Emitter emits the charges whereas collector collects the charges. Thus the width of collector is large compared to emitter. The base has smallest width, thus it is narrow.
- These 3 terminals together form pnp or npn transistor.

Transistor Current components



The above figure shows the various current components

of a pnp transistor across the forward and reverse biased junction. i.e., EB junction is forward biased and CB junction is reverse biased.

The current I_E flows in to the emitter and current I_C flows out of collector.

The emitter doping is much larger than that of base. This ensures that emitter current is formed entirely of holes and these all will flow in to the collector with a few holes recombining in the base.

→ The current due to holes in emitter = I_{Ep} ①

→ Hole current at collector side = I_{Ep} ②

→ Recombination of holes in the base gives the current, $I_{Ep} - I_{Ep}$ ③

→ The e^- lost in the base due to recombination has to be resupplied externally. This current is I_B ④

→ Some e^- will be injected from n to p in forward biased emitter junction. Current due to this electron is I_{En} ⑤

→ Some e^- are swept in to the base at the reverse biased collector junction due to thermal generation at the collector = I_{Co} ⑥

→ I_{Co} consist of both holes moving from n to p (I_{pco}) and e^- moving from p to n (I_{nco}).

$$I_{Co} = I_{pco} + I_{nco}$$

Basic Performance Parameters:

1. Emitter efficiency or injection efficiency (γ)

It is the effectiveness of emitting charge carriers from E to B.

$$\gamma = \frac{\text{Injected carrier current at EB Junction}}{\text{Total emitter current}}$$

$$\gamma = \frac{I_{EP}}{I_{EP} + I_{EN}}$$

2. Base Transport factor (B)

Effectiveness of base in transporting charge carriers injected from the emitter to the collector through the base. A better transport factor means less loss of charge carriers in the base.

$$B = \frac{\text{Injected carrier current reaching at CB Junction}}{\text{Injected carrier current at EB Junction.}}$$

$$B = \frac{I_{CP}}{I_{EP}}$$

$B = 1$ or 100% means no recombination in base.

3. Common Base current gain (α) or current transfer ratio or Emitter to collector current amplification

$$\begin{aligned}\alpha &= \frac{I_C}{I_E} = \frac{I_{CP}}{I_E} = \frac{B I_{EP}}{I_{EP} + I_{EN}} \\ &= B \cdot \frac{I_{EP}}{I_{EP} + I_{EN}} = B\gamma\end{aligned}$$

$$\text{i.e., } \alpha = B\gamma$$

Also called current transfer ratio.

4. Common Emitter current gain (β)

or base to collector current amplification

$$\begin{aligned}\beta &= \frac{I_C}{I_B} = \frac{I_{CP}}{I_B} \\ &= \frac{B \cdot I_{EP}}{I_{EN} + I_{EP} - I_{CP}} \\ &= \frac{B I_{EP}}{I_{EN} + I_{EP} - B I_{EP}}\end{aligned}$$

Divide Nr & Dr
by $I_{EN} + I_{EP}$,

$$\begin{aligned}\beta &= \frac{B \cdot I_{EP} / I_{EP} + I_{EN}}{\frac{I_{EN} + I_{EP}}{I_{EP} + I_{EN}} - \frac{B I_{EP}}{I_{EP} + I_{EN}}} \\ &= \frac{B \cdot \gamma}{1 - B \gamma} = \frac{\alpha}{1 - \alpha}\end{aligned}$$

$$\boxed{\beta = \frac{\alpha}{1 - \alpha}}$$

$$\beta(1 - \alpha) = \alpha$$

$$\beta - \alpha\beta = \alpha$$

$$\beta = \alpha + \alpha\beta$$

$$\beta = \alpha(1 + \beta)$$

$$\boxed{\alpha = \frac{\beta}{1 + \beta}}$$

Transistor action

1. Switching
2. Amplification.

Transistor as a switch.

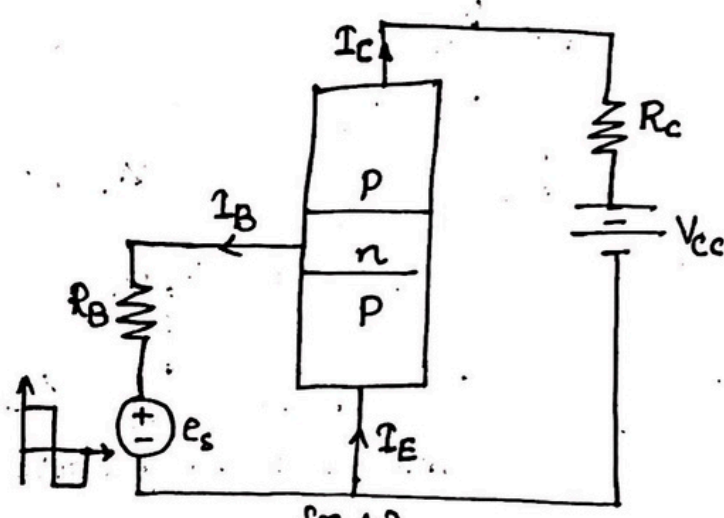


fig (1)

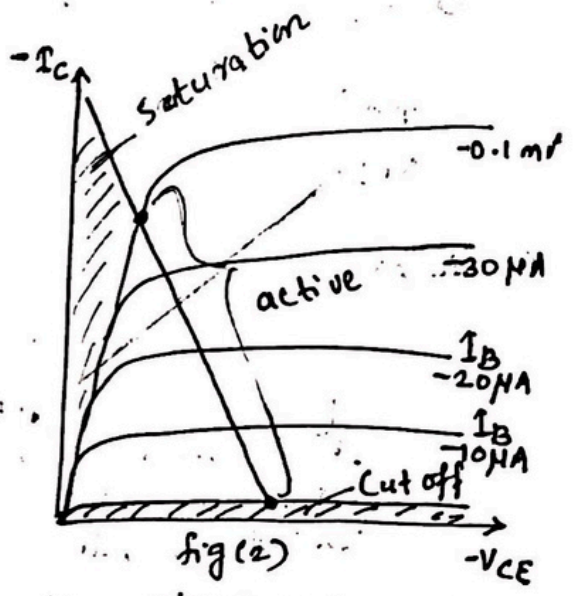


fig (2)

Transistor works as a switch i.e., either ON or OFF.

i.e., when switch is ON, short ckt

when switch is OFF, open ckt

We need to switch the device from one state to another and vice versa.

Cut OFF

Emitter Base and Collector Base junction are reverse biased.

$$\Delta P_E = P_n \cdot (e^{\frac{2V_{EB}}{KT}} - 1) \quad \text{--- (1)}$$

When $V_{EB} \ll \frac{KT}{2}$

$$\text{(1)} \Rightarrow \Delta P_E = -P_n \quad \text{--- (2)}$$

$$\Delta P_C = P_n (e^{\frac{2V_{CB}}{KT}} - 1) \quad \text{--- (3)}$$

When $V_{CB} \ll \frac{KT}{2}$

$$\text{(3)} \Rightarrow \Delta P_C = -P_n \quad \text{--- (4)}$$

Figure (3) shows hole distribution in base during cut off

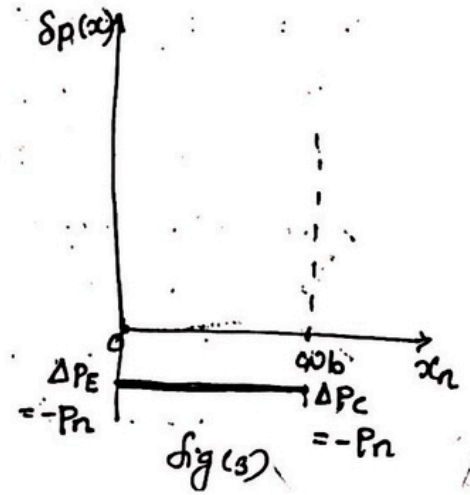


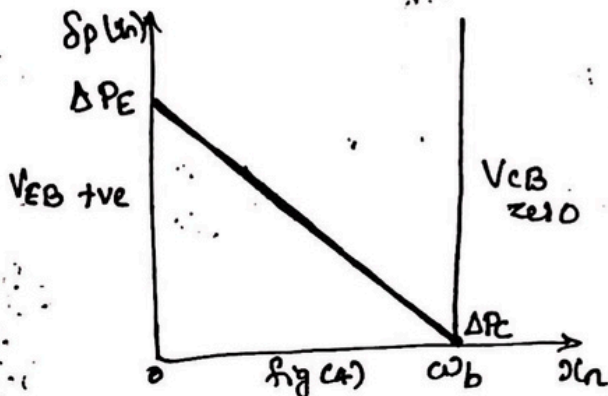
fig (3)

Saturation:

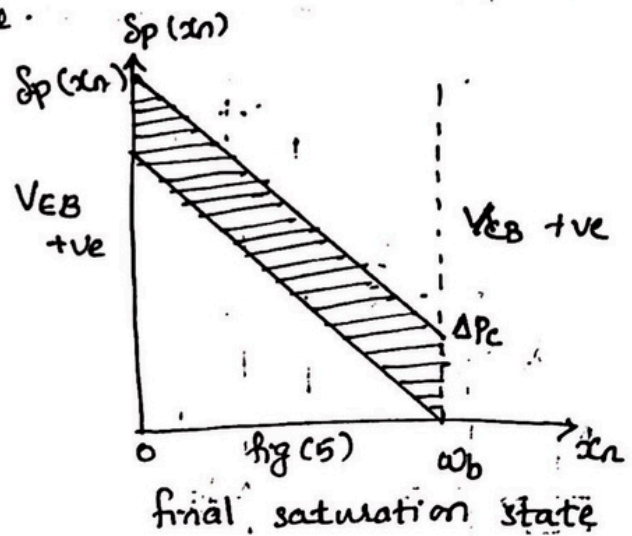
EB and CB are forward biased.

→ Saturation occurs when reverse bias across the CB junction reduced i.e., $V_{EB} \uparrow$

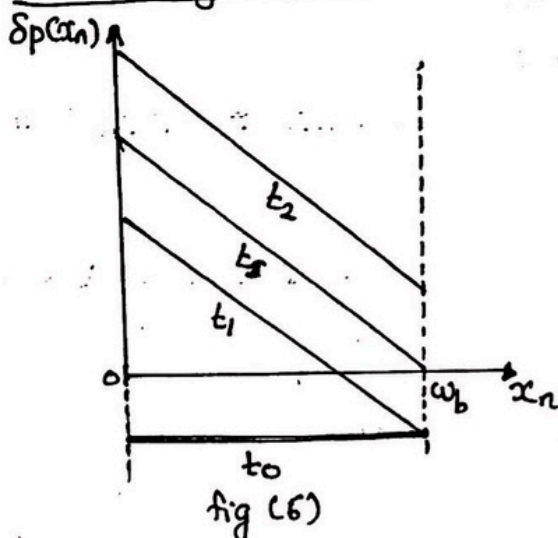
→ V_{CB} becomes +ve value.



Device has just reached saturation



Switching cycle



$t_0 \rightarrow$ cut off

$t_1 \rightarrow$ Normal active region

$t_s \rightarrow$ Beginning of saturation

$t_2 \rightarrow$ Final saturation state

→ As step input is given to the base, initially if the device is at cut off, excess hole distribution $Sp(xn) \approx 0$ or -ve value ($-P_n$).

→ As step input is given, I_B increases, hole distribution increases from t_0 to t_1 . (fig 6)

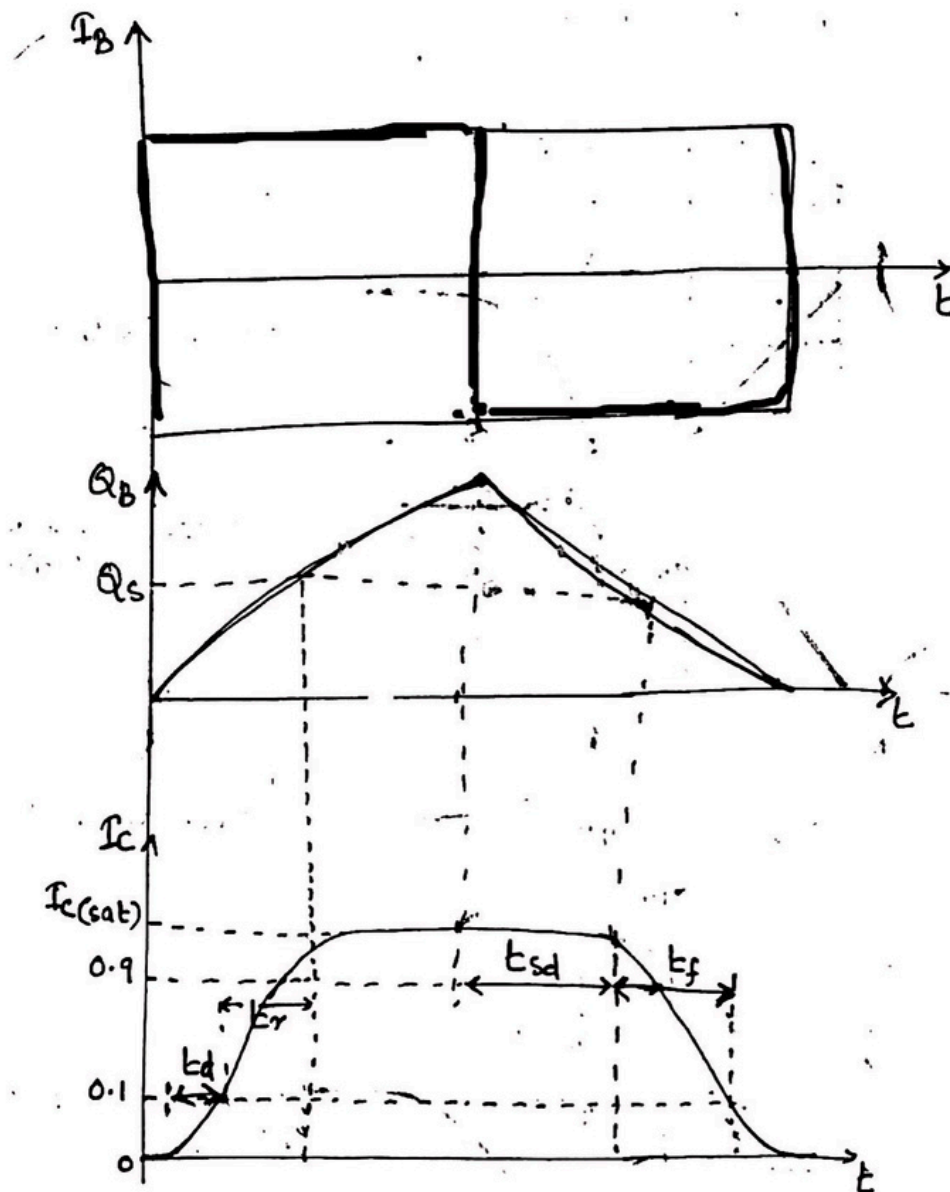
→ At time t_s , device enters saturation and finally hole distribution reaches final state t_2 .

→ The stored charge in base (Q_b) has increased, I_c increases and becomes $I_c \approx \frac{V_{cc}}{R_c}$

→ There is an exponential increase in I_c when charge rises from Q_b to Q_s at t_s .

→ When base is switched -ve, stored charge begins to decrease

→ When $Q_b > Q_s$, I_c remains constant for a stored delay time t_{sd} , after which I_c begins to fall. When $Q_b < Q_s$, I_c drops exponentially, when all the stored charge is removed, transistor moves to cut off.



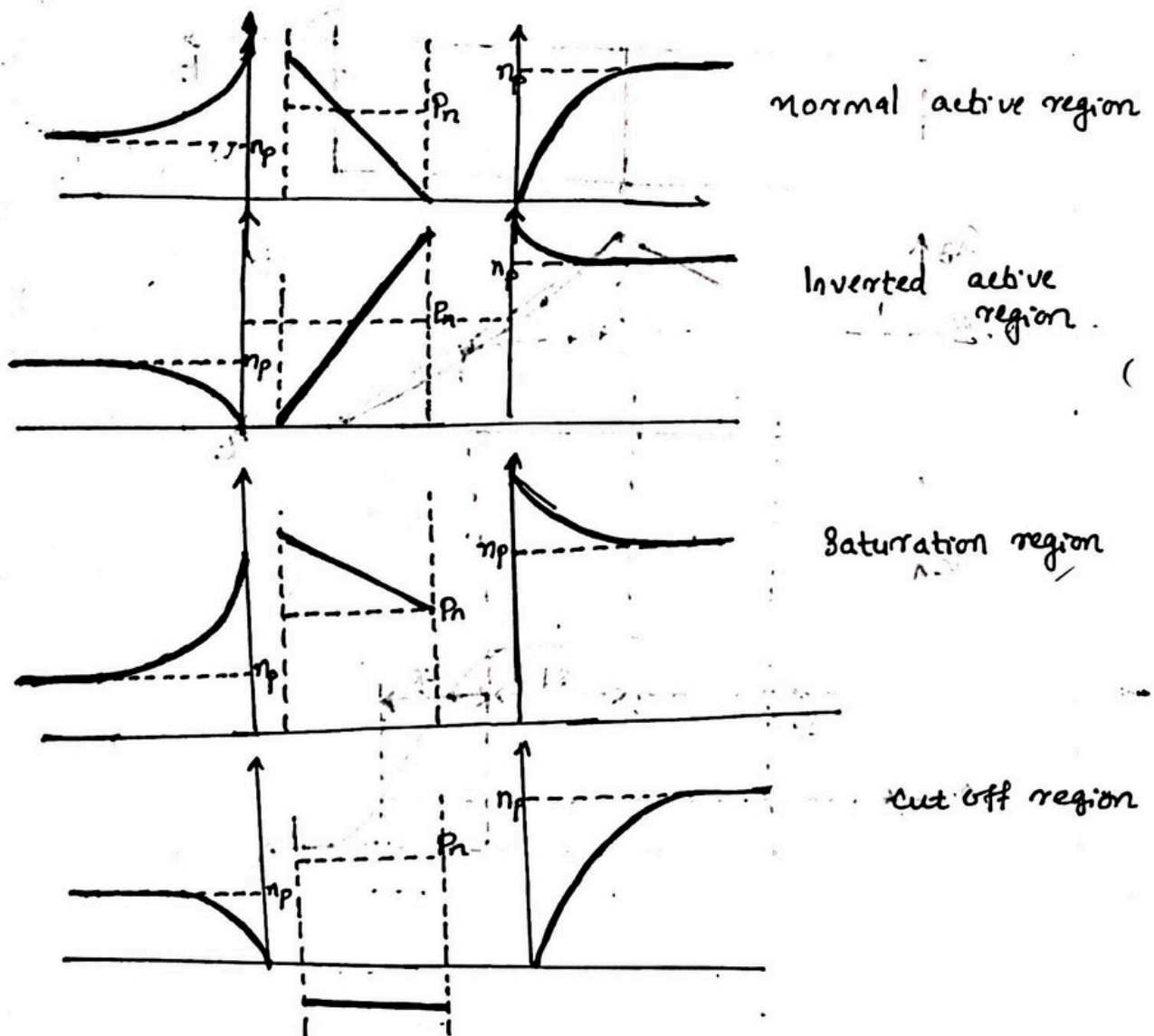
Delay time: The time period when I_c $\uparrow$ s from 0 to 10% of its final value.

Rise time (t_{rr}): The time period when I_c $\uparrow$ s from 10 to 90% of its final value.

Storage time (t_{sd}): The time period when I_c decreases from maximum value to 90% of final value.

Fall time (t_f): The time period when I_c $\downarrow$ s from 90 to 10% of its final value.

* Minority carrier distribution



Minority carrier distribution and terminal currents

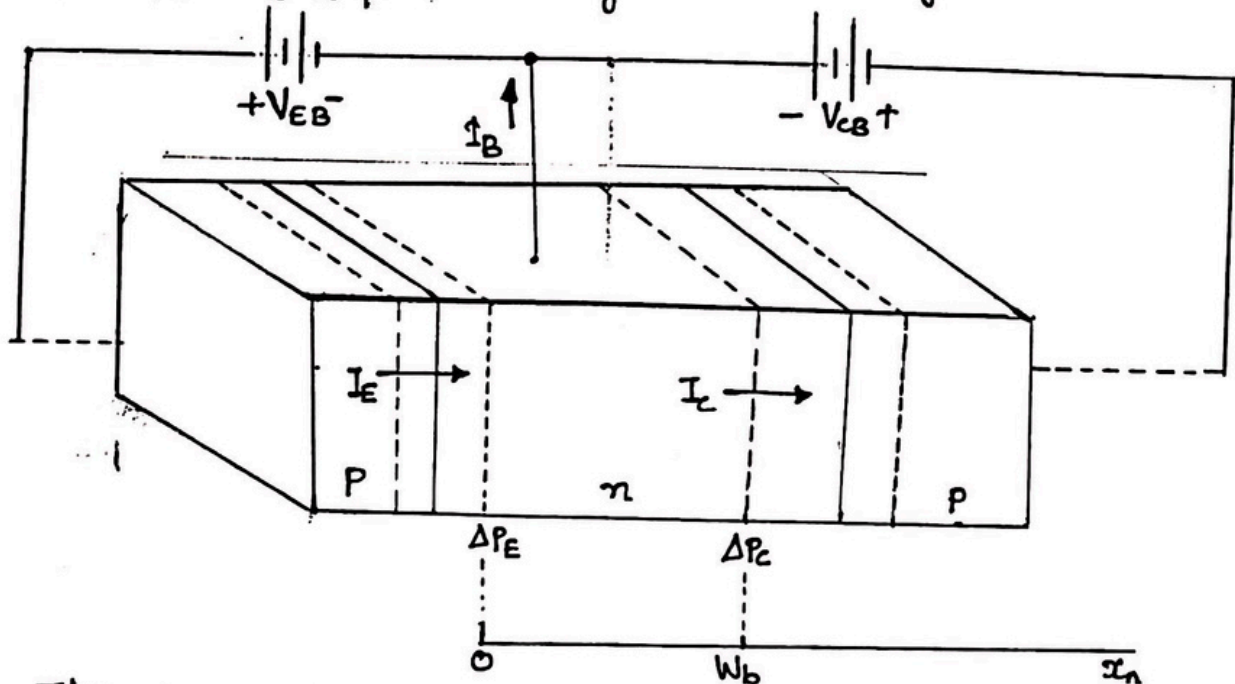
Assume holes are injected into the base region at the forward biased junction and these holes diffuse to collector junction.

Assumptions

1. Holes diffuse from Emitter to Collector.
2. Emitter current is made of holes, $\alpha = 1$
3. I_{CO} is negligible
4. All currents and voltages are steady state
5. The active part of the base and two junctions are of uniform cross sectional area A . current flow in the base is essentially one dimensional from E to C

Minority carrier distribution (solution of diffusion eqn in the base region)

Consider a pnp transistor. The recombination in the 2 depletion regions are neglected.



The current entering the base at the emitter junction = I_E
The hole current leaving the base at the collector = I_C

→ Base width = w_b

→ The excess hole concentration at the edge of emitter depletion = ΔP_E

→ Excess hole concentration at the collector side of base = ΔP_C

From diode eqn, we have,

$$\Delta P_n = P_n (e^{qV/KT} - 1)$$

So $\Delta P_E = P_n (e^{2V_{EB}/KT} - 1)$ ——— ①

$$\Delta P_C = P_n (e^{2V_{CB}/KT} - 1)$$
 ——— ②

If $V_{EB} \gg \frac{KT}{q}$ ie, strong FB

$$\text{①} \Rightarrow \Delta P_E = P_n \cdot e^{2V_{EB}/KT}$$
 ——— ③

If $V_{CB} \ll 0$, strong RB

$$\text{②} \Rightarrow \Delta P_C = -P_n$$
 ——— ④

The eqn for excess carriers (from continuity eqn)

$$\frac{\partial^2 \delta p(x_n)}{\partial x_n^2} = \frac{\delta p(x_n)}{L_p} \quad \text{soln to this eqn is}$$

$$\delta p(x_n) = C_1 \cdot e^{x_n/L_p} + C_2 \cdot e^{-x_n/L_p}$$
 ——— ⑤

where L_p is the diffusion length for hole's.

Apply boundary conditions,

In ⑤, When $x_n = 0$, $\delta p(x_n = 0) = C_1 + C_2 = \Delta P_E$ ——— ⑥

When $x_n = w_b$, $\delta p(x_n = w_b) = C_1 \cdot e^{w_b/L_p} + C_2 \cdot e^{-w_b/L_p} = \Delta P_C$ ——— ⑦

Solving for C_1 and C_2

From (6), $C_1 = \Delta P_E - C_2$ — (8)

Substitute (8) in (7)

$$\Delta P_C = (\Delta P_E - C_2) e^{wb/Lp} + C_2 \cdot e^{-wb/Lp}$$

$$\Delta P_C = \Delta P_E e^{wb/Lp} - C_2 \cdot e^{wb/Lp} + C_2 e^{-wb/Lp}$$

$$\Delta P_C = \Delta P_E \cdot e^{wb/Lp} - C_2 \left(e^{wb/Lp} - e^{-wb/Lp} \right)$$

$$\Delta P_E \cdot e^{wb/Lp} - \Delta P_C = C_2 \left(e^{wb/Lp} - e^{-wb/Lp} \right)$$

$$C_2 = \frac{\Delta P_E \cdot e^{wb/Lp} - \Delta P_C}{e^{wb/Lp} - e^{-wb/Lp}} \quad \text{--- (9)}$$

$$\begin{aligned} (8) \Rightarrow C_1 &= \Delta P_E - C_2 \\ &= \Delta P_E - \frac{\Delta P_E \cdot e^{wb/Lp} - \Delta P_C}{e^{wb/Lp} - e^{-wb/Lp}} \end{aligned}$$

$$C_1 = \frac{\Delta P_E \left(e^{wb/Lp} - e^{-wb/Lp} \right) - \Delta P_E \cdot e^{wb/Lp} + \Delta P_C}{e^{wb/Lp} - e^{-wb/Lp}}$$

$$C_1 = \frac{\Delta P_C - \Delta P_E \cdot e^{-wb/Lp}}{e^{wb/Lp} - e^{-wb/Lp}} \quad \text{--- (10)}$$

Substitute (9) and (10) in (5)

$$\delta p(x_n) = C_1 e^{x_n/Lp} + C_2 \cdot e^{-x_n/Lp}$$

$$= \frac{\Delta P_C - \Delta P_E \cdot e^{-wb/Lp}}{e^{wb/Lp} - e^{-wb/Lp}} \cdot e^{x_n/Lp} + \frac{\Delta P_E \cdot e^{wb/Lp} - \Delta P_C}{e^{wb/Lp} - e^{-wb/Lp}} \cdot e^{-x_n/Lp}$$

$$\delta p(x_n) = \frac{\Delta P_c \cdot e^{x_n/L_p} - \Delta P_E \cdot e^{-w_b/L_p} \cdot e^{x_n/L_p} + \Delta P_E \cdot e^{w_b/L_p} \cdot e^{-x_n/L_p} - \Delta P_c \cdot e^{-x_n/L_p}}{e^{w_b/L_p} - e^{-w_b/L_p}}$$

$$\delta p(x_n) = \frac{\Delta P_c \left[e^{x_n/L_p} - e^{-x_n/L_p} \right] - \Delta P_E \left[e^{-w_b/L_p} \cdot e^{x_n/L_p} - e^{w_b/L_p} \cdot e^{-x_n/L_p} \right]}{e^{w_b/L_p} - e^{-w_b/L_p}} \quad \text{--- (11)}$$

If collector x_n has strong reverse bias and P_n is negligible compared to ΔP_E .

ΔP_c is negligible compared to ΔP_E

$$\therefore \Delta P_c \approx 0$$

$$\textcircled{11} \Rightarrow \delta p(x_n) = \frac{\Delta P_E \left[e^{w_b/L_p} \cdot e^{-x_n/L_p} - e^{-w_b/L_p} \cdot e^{x_n/L_p} \right]}{e^{w_b/L_p} - e^{-w_b/L_p}} \quad \text{--- (12)}$$

Evaluation of terminal currents:

We have,

$$I_p(\text{diff}) = -q A D_p \cdot \frac{d}{dx_n} p(x)$$

$$= -q A D_p \cdot \frac{d}{dx_n} (P_0 + \delta p)$$

$$I_p(x_n) = -q A D_p \cdot \frac{\partial}{\partial x_n} \delta p(x_n) \quad \text{--- (13)}$$

Substitute eqn (5) in (13)

$$\therefore I_p(x_n) = -q A D_p \cdot \frac{\partial}{\partial x_n} \left[c_1 \cdot e^{x_n/L_p} + c_2 \cdot e^{-x_n/L_p} \right]$$

$$= -q A D_p \cdot \left[\frac{1}{L_p} \cdot c_1 \cdot e^{x_n/L_p} - \frac{1}{L_p} \cdot c_2 \cdot e^{-x_n/L_p} \right]$$

$$I_p(x_n) \Rightarrow -\frac{q A D_p}{L_p} \left[c_1 e^{x_n/L_p} - c_2 e^{-x_n/L_p} \right] \quad \text{--- (14)}$$

To find I_{Ep} , Put $x_n = 0$ in eqn (14)

$$I_{Ep} = -\frac{2AD_p}{L_p} [C_1 - C_2]$$

$$I_{Ep} = \frac{2AD_p}{L_p} [C_2 - C_1] \quad (14)$$

Substitute (9) and (10) in (14)

$$I_{Ep} = \frac{2AD_p}{L_p} \left[\frac{\Delta P_E \cdot e^{\frac{wb}{L_p}} - \Delta P_C}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} - \frac{\Delta P_C - \Delta P_E \cdot e^{-\frac{wb}{L_p}}}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right]$$

$$= \frac{2AD_p}{L_p} \left[\frac{\Delta P_E (e^{\frac{wb}{L_p}} + e^{-\frac{wb}{L_p}}) - 2\Delta P_C}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right]$$

$$= \frac{2AD_p}{L_p} \left[\frac{\Delta P_E (e^{\frac{wb}{L_p}} + e^{-\frac{wb}{L_p}})}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} - \frac{2\Delta P_C}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right]$$

$$= \frac{2AD_p}{L_p} \left[\frac{\Delta P_E \cdot 2\cosh\left(\frac{wb}{L_p}\right)}{2\sinh\left(\frac{wb}{L_p}\right)} - \frac{2\Delta P_C}{2\sinh\left(\frac{wb}{L_p}\right)} \right]$$

$$I_{Ep} = \frac{2AD_p}{L_p} [\Delta P_E \cdot \coth\left(\frac{wb}{L_p}\right) - \Delta P_C \cdot \operatorname{cosech}\left(\frac{wb}{L_p}\right)] \quad (15)$$

To find I_{cp} , Put $x_n = w_b$ in (14)

$$I_{cp} = -\frac{2AD_p}{L_p} [C_1 \cdot e^{\frac{wb}{L_p}} - C_2 \cdot e^{-\frac{wb}{L_p}}]$$

$$= \frac{2AD_p}{L_p} [C_2 \cdot e^{-\frac{wb}{L_p}} - C_1 \cdot e^{\frac{wb}{L_p}}]$$

$$I_{cp} = \frac{2AD_p}{L_p} \left[\left(\frac{\Delta P_E \cdot e^{\frac{wb}{L_p}} - \Delta P_C}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right) e^{-\frac{wb}{L_p}} - \left(\frac{\Delta P_C - \Delta P_E \cdot e^{-\frac{wb}{L_p}}}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right) e^{\frac{wb}{L_p}} \right]$$

$$\begin{aligned}
 I_{cp} &= \frac{2AD_p}{L_p} \left[\frac{\Delta P_E \cdot e^{\frac{wb}{L_p}} \cdot e^{-\frac{wb}{L_p}} - \Delta P_C \cdot e^{-\frac{wb}{L_p}}}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} - \frac{\Delta P_C \cdot e^{\frac{wb}{L_p}} - \Delta P_E \cdot e^{-\frac{wb}{L_p}}}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right] \\
 &= \frac{2AD_p}{L_p} \left[\frac{\Delta P_E - \Delta P_C \cdot e^{-\frac{wb}{L_p}}}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} - \frac{\Delta P_C \cdot e^{\frac{wb}{L_p}} - \Delta P_E}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right] \\
 &= \frac{2AD_p}{L_p} \left[\frac{2\Delta P_E - \Delta P_C (e^{\frac{wb}{L_p}} + e^{-\frac{wb}{L_p}})}{e^{\frac{wb}{L_p}} + e^{-\frac{wb}{L_p}}} \right] \\
 &= \frac{2AD_p}{L_p} \left[\frac{2\Delta P_E}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} - \frac{\Delta P_C \cdot (e^{\frac{wb}{L_p}} + e^{-\frac{wb}{L_p}})}{e^{\frac{wb}{L_p}} - e^{-\frac{wb}{L_p}}} \right] \\
 &= \frac{2AD_p}{L_p} \left[\frac{2\Delta P_E}{2 \sinh(wb/L_p)} - \frac{\Delta P_C \cdot 2 \cosh(wb/L_p)}{2 \sinh(wb/L_p)} \right]
 \end{aligned}$$

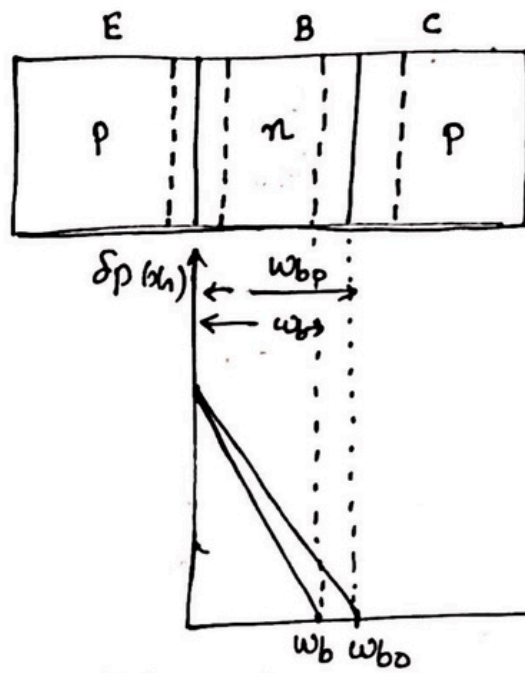
$$I_{cp} = \frac{2AD_p}{L_p} \left[\Delta P_E \cdot \operatorname{cosech}(wb/L_p) - \Delta P_C \cdot \coth(wb/L_p) \right] \quad (16)$$

$$I_B = I_E - I_C$$

$$\begin{aligned}
 &= \frac{2AD_p}{L_p} \left[\Delta P_E \cdot \coth(wb/L_p) - \Delta P_C \cdot \operatorname{cosech}(wb/L_p) \right] \\
 &\quad - \frac{2AD_p}{L_p} \left[\Delta P_E \cdot \operatorname{cosech}(wb/L_p) - \Delta P_C \cdot \coth(wb/L_p) \right] \\
 &= \frac{2AD_p}{L_p} \left[\Delta P_E \{ \coth(wb/L_p) - \operatorname{cosech}(wb/L_p) \} + \right. \\
 &\quad \left. + \Delta P_C \{ \coth(wb/L_p) - \operatorname{cosech}(wb/L_p) \} \right]
 \end{aligned}$$

$$I_B = \frac{2AD_p}{L_p} \left[(\Delta P_E + \Delta P_C) (\coth(wb/L_p) - \operatorname{cosech}(wb/L_p)) \right] \quad (17)$$

Base width Modulation (Early effect)



The effective base width ' w_b ' of the base region is the difference between the total base width w_{b0} and the depletion layer width of the collector base junction in to the base region (neglecting the depletion layer width of the emitter base junction).

The effective base width decreases with increases in reverse bias of the CB junction. This is called early effect or base width modulation.

Due to base width modulation, as reverse bias on the collector base junction increases, I_c and I_E increases due to increase in V_{CB} , w_b decreases which increases the slope of minority carrier distribution. So as the slope increases, I_{Ep} and I_{Ep} increases.

As width of the base reduced, the transport factor B increases, recombination in base reduces reducing I_B . As B increases, α increases, and β increases.

$$I_E, I_c \uparrow, I_{Ep}, I_{cp} \uparrow$$

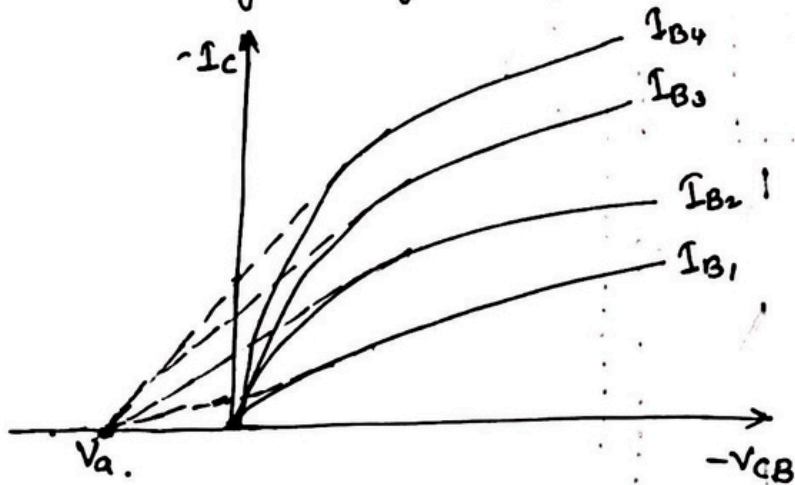
$$B \uparrow$$

$$w_b \downarrow$$

$$I_B \downarrow$$

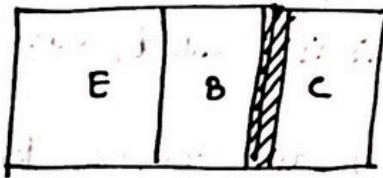
$$\alpha, \beta \uparrow$$

If the o/p characteristics is extrapolated to meet the voltage axis, all the curves meet at the same voltage called early voltage (V_a)

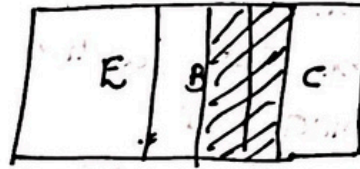


Punch through effect

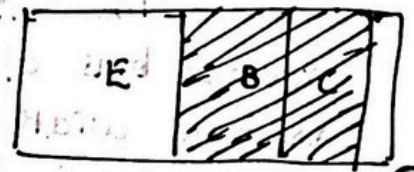
If V_{cb} is increased to a large value, such that w_b decreases to the extent that the collector depletion layer fills the entire base. This is called punch through effect. Now, holes can be directly swept from E to C and transistor action is lost (This has to be avoided)



Small R_B



Large R_B

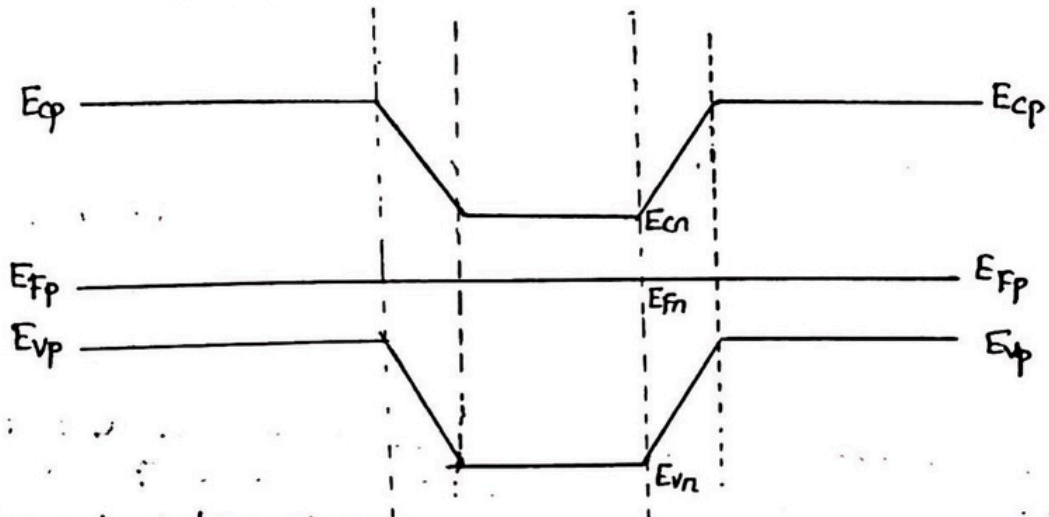


Punch through effect

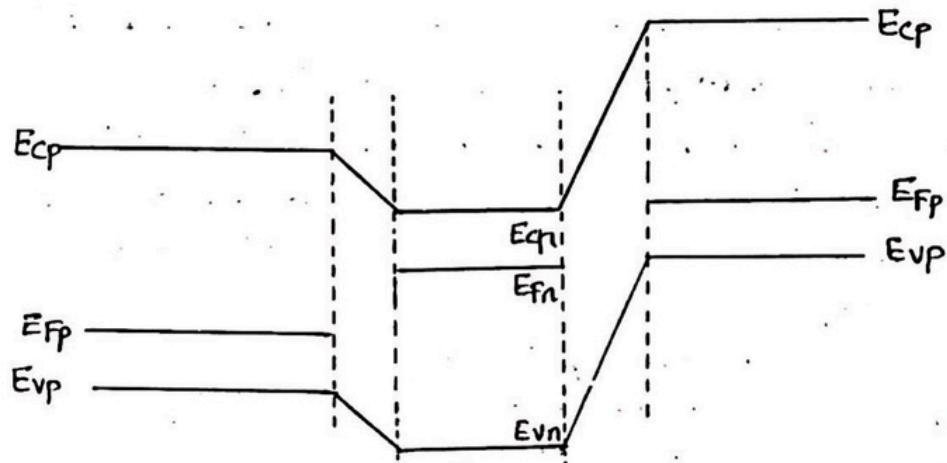
Energy band diagram.

U

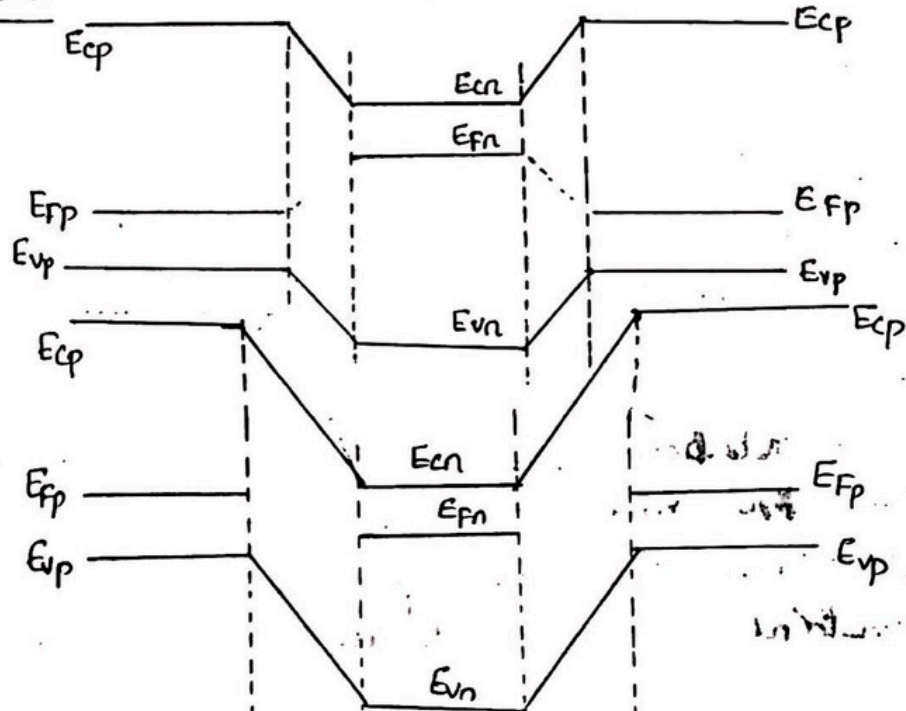
1) Equilibrium



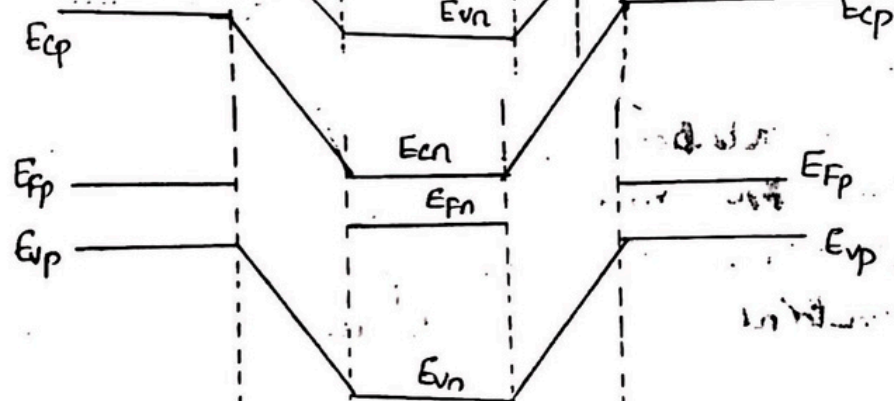
2) Normal active region



3) Saturation



4) Cut off



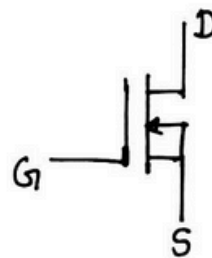
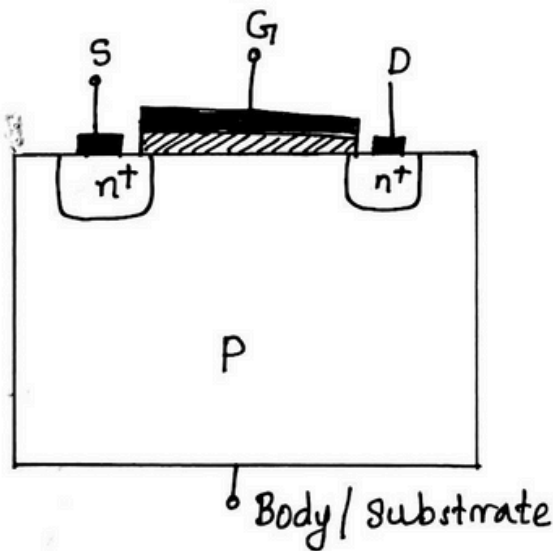
MODULE 4

Metal Oxide semiconductor FET (MOSFET)

MOSFET $\begin{cases} \rightarrow n \text{ channel} \\ \rightarrow p \text{ channel} \end{cases}$

A MOSFET has terminals Source (S), Drain (D), Gate (G) and a fourth terminal called body or substrate.

MOS : N-channel MOSFET consist of a P type substrate over which n^+ is heavily doped. A thin oxide layer separates the conducting gate from Si surface. No current flows from D to S without a conducting n-channel between them.

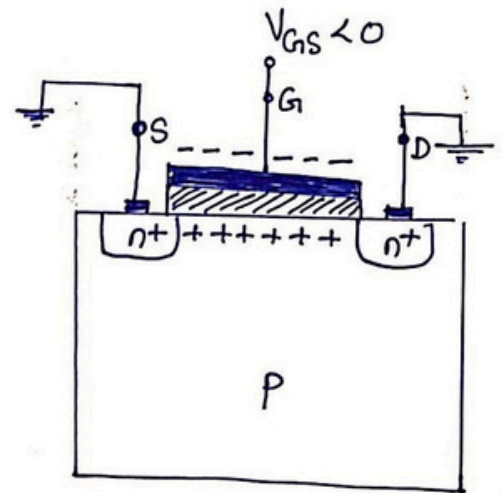


- MOS is a majority carrier device.
- The current through the conducting channel between S and D is controlled by voltage applied on the gate.

Working of n-MOS

1. When $-ve$ voltage is applied on the gate i.e., $V_{GS} < 0$

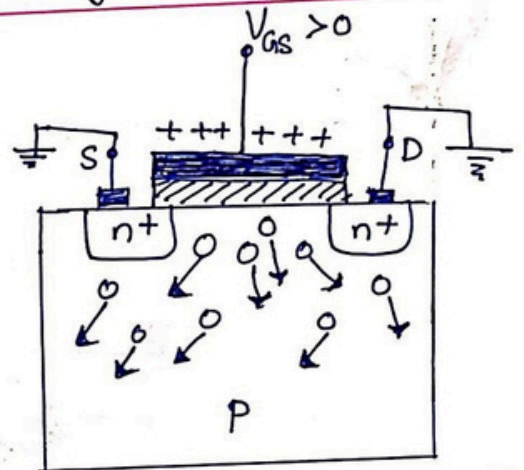
Due to $-ve$ input at the gate, $-ve$ charges are deposited on the gate metal and $+ve$ charges are induced in the substrate. This is called ACCUMULATION of charge carriers beneath the gate. This accumulation is due to the attraction of $+ve$ charges by the $-ve$ charges deposited on the gate.



2. When $+ve$ voltage is applied at the gate: i.e., $V_{GS} > 0$

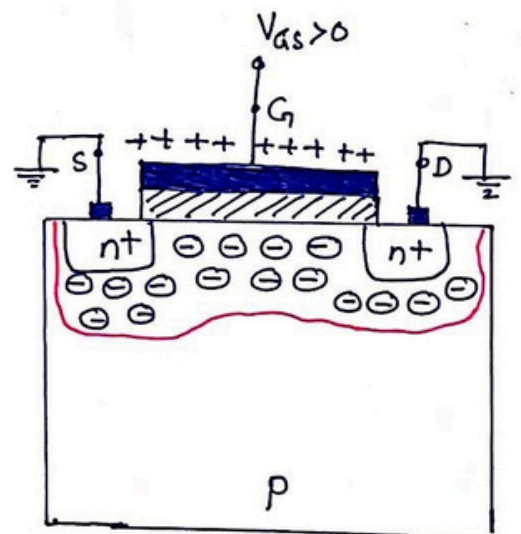
When $+ve$ voltage is applied on the gate, they repel the holes in the P substrate.

→ This causes the $-ve$ ions to be left behind pushing the holes away.



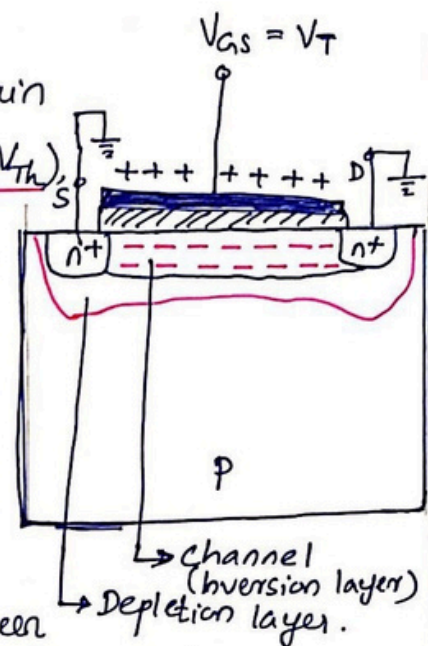
→ Thus the region beneath the gate is depleted of mobile charge carriers, and forms

DEPLETION REGION. This region is depleted of mobile charge carriers.



3. Higher +ve voltage applied on the gate

As +ve voltage increases, more and more holes inside P substrate repels away. When +ve voltage exceeds certain voltage called threshold voltage (V_T or V_{Th}), the large +ve gate voltage begins to attract the minority electrons in the P substrate and e^- s in the n^+ regions towards it.



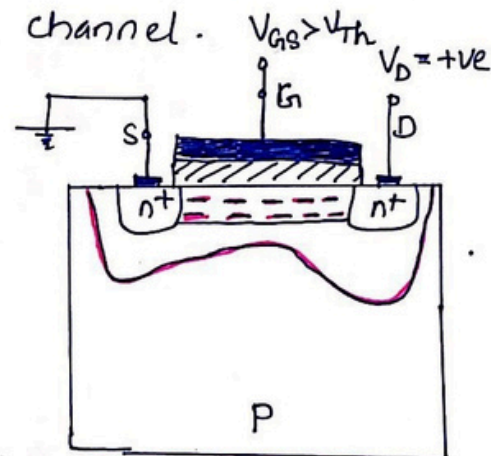
→ The e^- s are attracted beneath the gate forming a conducting path between S and D. and this path is called CHANNEL. and it is said to be inverted (Inversion Layer).

→ Once the channel is formed, current flow starts when V_{DS} is applied.

Threshold voltage is the minimum gate source voltage required to induce the channel. $V_{GS} > V_{Th}$

4. $V_{GS} > V_{Th}$ and $V_{DS} \uparrow$

As V_{DS} increases, more and more electrons are attracted towards the drain terminal. So Drain current (I_D) flows from D to S.



$V_D < V_G - V_T$
Linear region.

→ The channel act as a resistor. The Ohmic voltage drop across the channel varies from '0' near source end to V_{GS} near drain end.

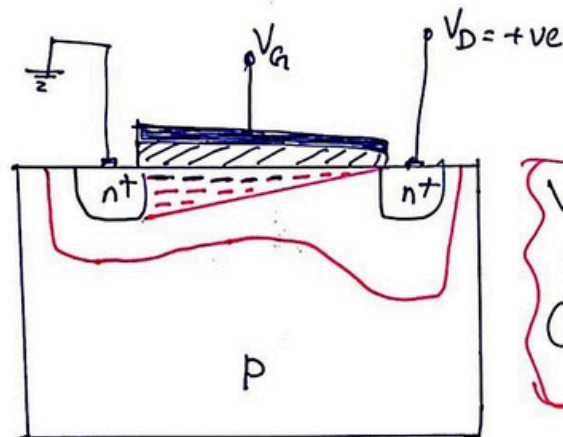
→ This region is called LINEAR / TRIODE REGION

5. V_{DS} increases again

Once the drain voltage is increased to the point that $V_G - V_D = V_T$, threshold is barely maintained near the drain end and the channel is said to be Pinched off.

ie, If V_D is increased, V_{th} becomes -ve ($V_G - V_D = V_{th}$)

→ When the channel is pinched off, device enters saturation.



$V_D = V_G - V_T$
Pinch off region
(onset of saturation)

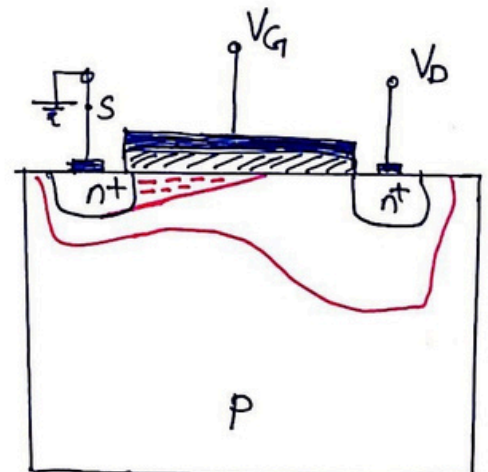
6. When $V_{DS} > V_{DS(sat)}$

Increasing the drain voltage beyond this point ($V_{DS(sat)}$) causes the point at which the channel gets pinched off to move more and more into the channel closer to source.
→ Due to high depletion region formed at drain end, a high longitudinal electric field is created across the channel.

→ e^- s flow from S to D.

→ Drain current is now said to be saturated and remains constant even if the V_{DS} is varied.

$V_D > V_{GS} - V_T$
Saturation region.

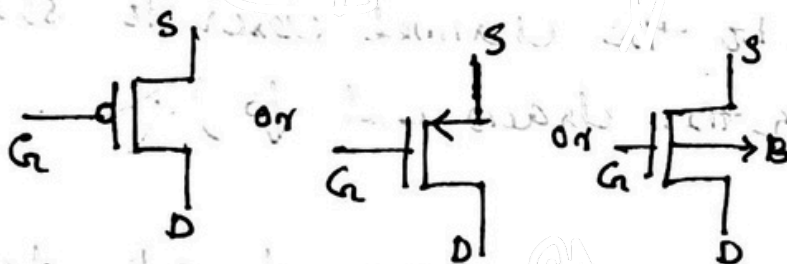


P-MOS (P channel MOS)

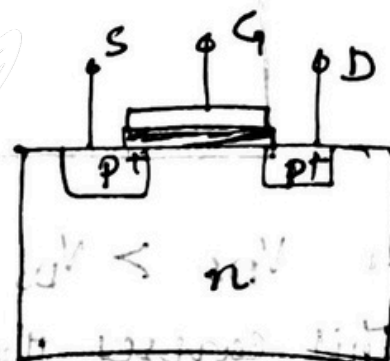
$P^+ \rightarrow$ source & drain.

$n \rightarrow$ substrate.

Symbol

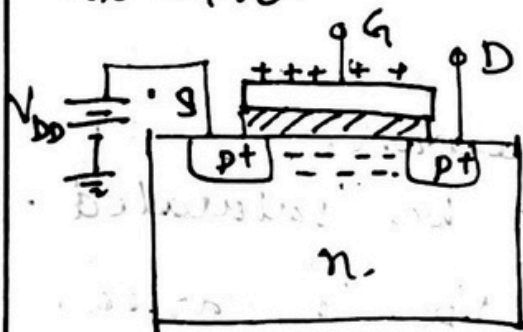


V_{DS}



Pmos

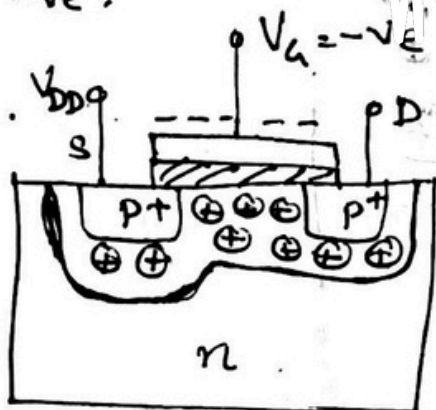
(1) $V_{GS} = +ve.$



$V_{GS} = +ve.$

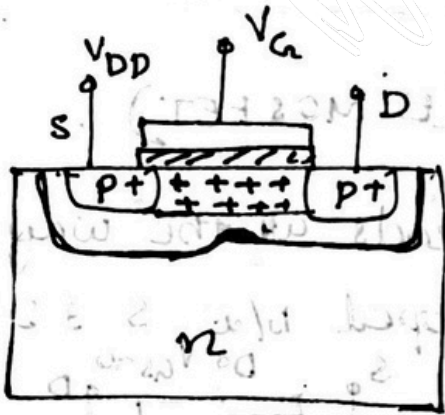
Accumulation of e^- beneath the gate.

(2) $V_{GS} = -ve.$



Gate repels e^- in n substrate. Depletion region is formed.

② $V_{GS} < V_{TH}$

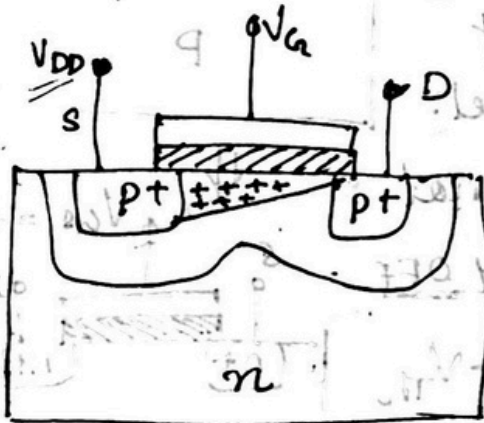


Channel is formed when:
 $V_{GS} = -ve$ & $V_{GS} < V_{TH}$.

Current I_D flows when a

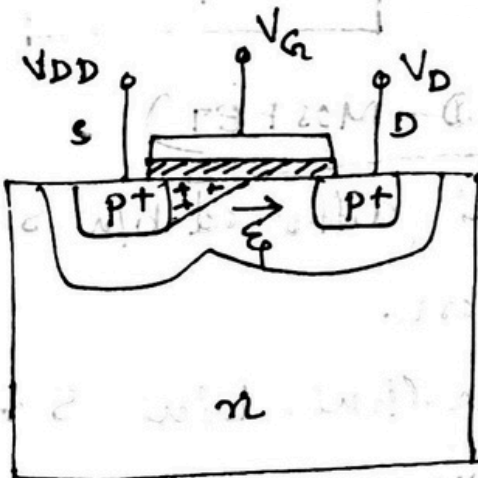
$V_{DS} = -ve$ voltage is applied as holes move from source to D, I_D also from S to D

④ $V_{DS} = V_{GS} - V_{TH}$ or $V_D = V_G - V_{TH}$



Pinch off occurs.

⑤

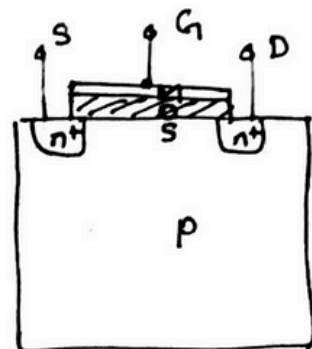


Saturation.

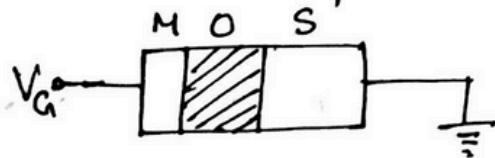
Ideal MOS Capacitor:

MOS capacitor is a two terminal Metal oxide semiconductor system.

consider nmos.



● Structure of MOS capacitor



* Band diagrams

a) At equilibrium, $V=0$ fig (i)

here $\Phi_m = \Phi_s$ so there is no band bending.

Φ_m : modified work function for the metal-oxide interface

Φ_s : Modified work function for the sc-oxide interface.

b) Accumulation $V < 0$ i.e. $V_G < 0$ fig (ii)

If a -ve voltage is applied between the metal and semiconductor, -ve charges are deposited on the metal, so +ve charges are accumulated on the sc surface.

Due to deposition of -ve charges on the metal, the Fermi level of metal is raised relative to E_F , so the material is more p type near the surface and bands bend upward.

$E_c(x) = \frac{1}{2} \frac{d}{dx} E_i(x)$ so the oxide layer bends

in the direction of electric field.

We have $P = P_i e^{(E_i - E_f)/kT}$ At sc surface as

$P \uparrow$ $E_i - E_f$ should increase,

but E_f is fixed, so E_i must increase.

Such a process in which majority carrier concentration increases near the surface than the bulk in a sc is called ACCUMULATION

(c) Depletion $V > 0$ fig (iii)

If a +ve voltage is applied from the metal  sc, +ve charges are deposited on the metal and this causes to move the fermi level of the metal downward compared to equilibrium.

Due to +ve charges on the metal, holes inside the sc are repelled away leaving behind -vely charged ions. This causes the formation of depletion layer near the surface and the surface becomes less p type. So E_i moves closer to E_f and the bands bend downwards.

d) Inversion $V \gg 0$ fig (iv)

If the +ve voltage is increased much further, the bands at the sc surface bends downwards more strongly. The +ve charge on the metal attracts the electrons (minority carriers) in the p type sc and the sc surface becomes n type compared to bulk. This causes the formation of channel on the sc.

Band diagrams - n MOS

fig (i)

a) Equilibrium, $V=0$

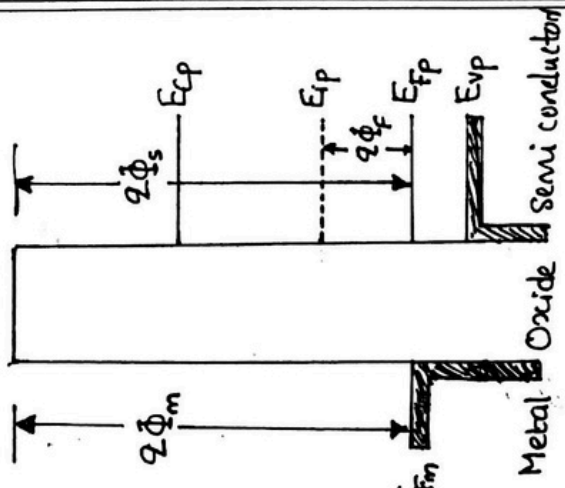


fig (ii)

b) Accumulation, $V < 0$

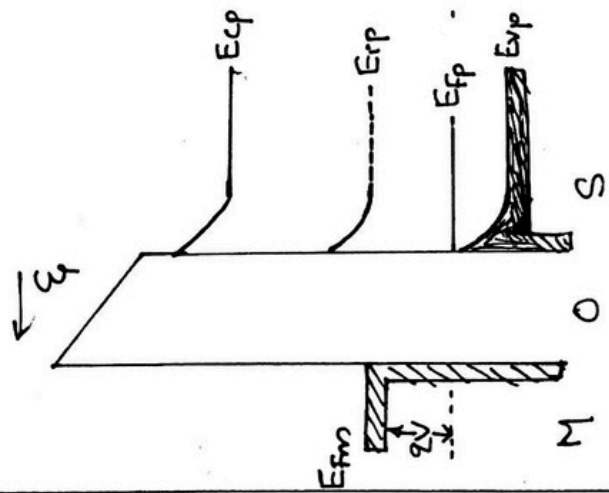


fig (iii)

c) Depletion, $V > 0$

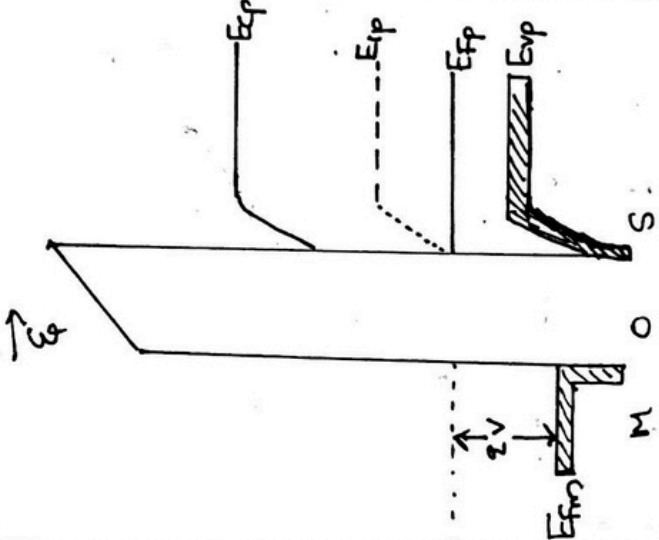
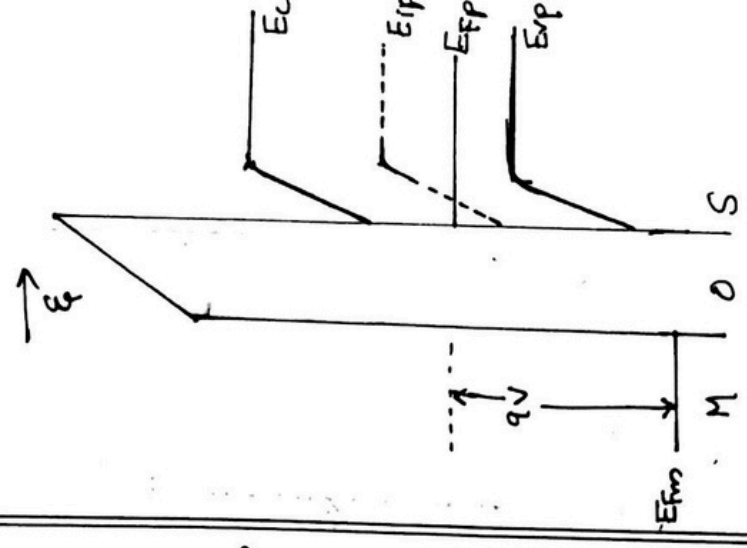


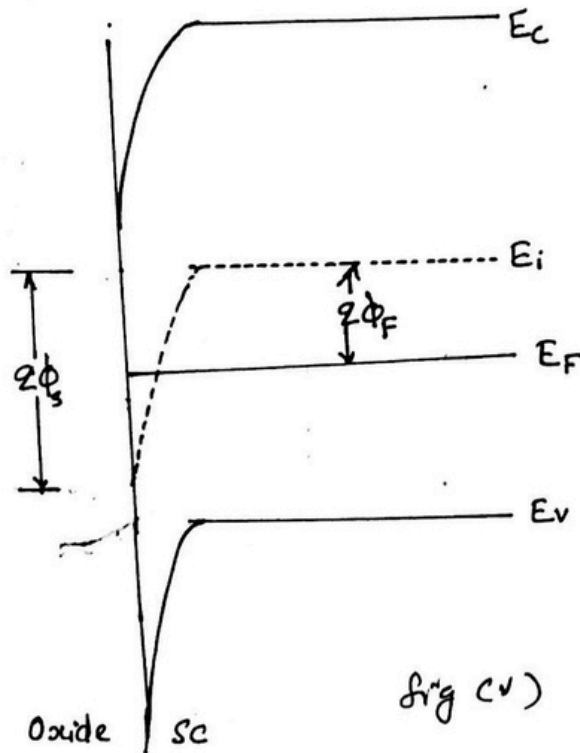
fig (iv)

d) Inversion, $V \gg 0$



Strong inversion: Surface is strongly n type when the material is p type and vice versa.

$$\phi_{s(\text{inv})} = 2\phi_F = 2 \frac{kT}{q} \ln \frac{N_A}{n_i}$$



$2\phi_s \rightarrow$ band bending at the surface

ϕ_s is called surface potential

when $\phi_s = 0$, flat band condition fig (i)

when $\phi_s < 0$, the bands bend up at the surface
ie, Accumulation fig (ii)

when $\phi_s > 0$, the bands bend downwards,
ie, Depletion fig (iii)

when $\phi_s > \phi_F$, bands bend downwards more.
ie, Inversion fig (iv)

when $\phi_s = 2\phi_F$ strong inversion fig (v)

Effects of real surfaces:

In real MOS system (eg:- $\text{Al-SiO}_2\text{-Si}$) the work function of metal and semiconductor may be different.

1. Work function difference:

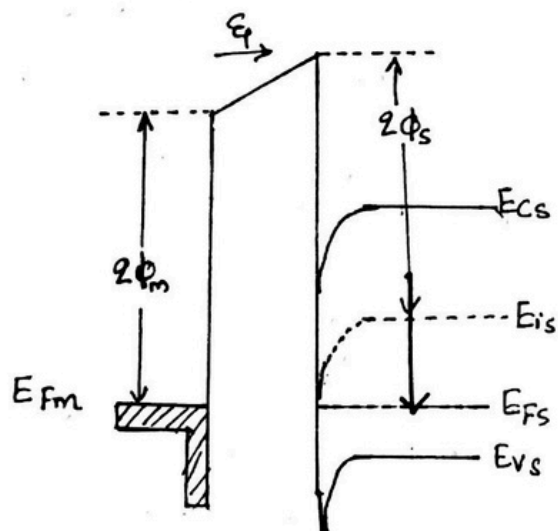
Let Φ_{ms} be the difference in work function of metal and semiconductor.

$$\Phi_{ms} = \Phi_m - \Phi_s \quad \text{which is always -ve.}$$

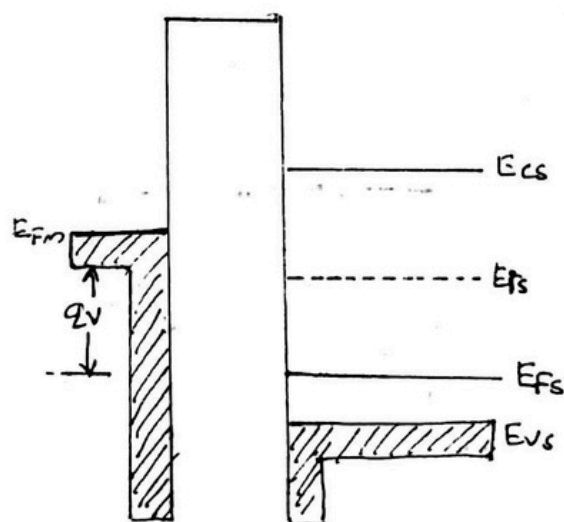
So at equilibrium, instead of flat bands, there would be band bending. This implies a tilt in the oxide layer and signifies the presence of electric field.

At equilibrium, metal is +vely charged and SC surface is -vely charged.

So for -ve Φ_{ms} , an inversion layer may exist in the SC under equilibrium. To obtain flat band condition, -ve voltage must be applied to the gate.



a) Equilibrium ($v=0$)



b) Flat band

b) Interface charge:

$$V = V_{FB} = \Phi_{ms}$$

In addition to the work function difference, the equilibrium MOS structure is affected by charges in the insulator and at the SC oxide interface.

The different types of charges in the oxide are as follows.

a) Mobile ionic charges (Q_m)

Alkali metals such as Na^+ , can move through crystalline SiO_2 which shift the threshold voltage of MOS system.

b) Fixed oxide charge (Q_f)

These are the charges located within the oxide as a sheet of +ve charge near Si-SiO₂ interface. These charges are formed during the oxidation process.

c) Interface trap charge (Q_{it})

These are charges present at the Si-SiO₂ interface due to the sudden termination of SC crystal at the interface.

d) Oxide trapped charges (Q_{ot})

The oxide may contain trapped charges within the defects presents in the oxide. Holes are trapped within the oxide during processing, whereas highly mobile e^- s move away from the oxide. Therefore Q_{ot} is a +ve charge.

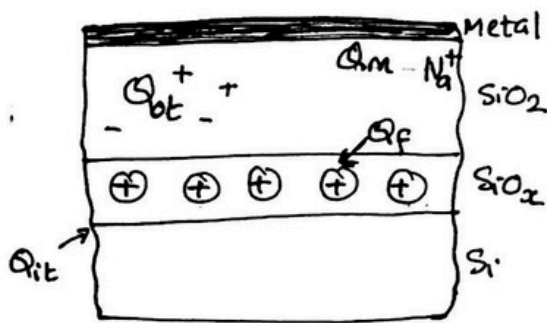


fig (a)

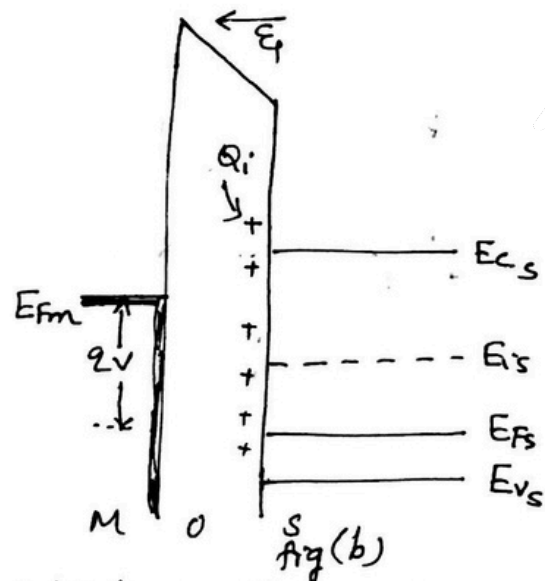


fig (b)

The above figure (b) shows the interface charges as an equivalent sheet of +ve charge Q_i at the oxide-SC interface. This +ve charge induces an equivalent -ve charge in the SC, which requires a -ve gate voltage to achieve the flat band condition.

So $V_{FB} = \phi_{ms} - \frac{Q_i}{C}$

(7)

Threshold voltage (V_T)

Minimum gate voltage required to form the channel (inversion layer) is called threshold voltage.

The voltage required to obtain depletion layer is added with the voltage required to obtain the inversion layer to get the threshold voltage.

$$V_{Th} = -\frac{Q_d}{C_{ox}} + \phi_s(\text{inv})$$

$$V_{Th} = -\frac{Q_d}{C_{ox}} + 2\phi_F \rightarrow \text{Ideal MOS}$$

Where $Q_d = -2N_a W_m$

$$\phi_F = \frac{KT}{q} \ln \frac{N_a}{n_i}$$

$$V_{FB} = \phi_{ms} - \frac{Q_i}{C_{ox}}$$

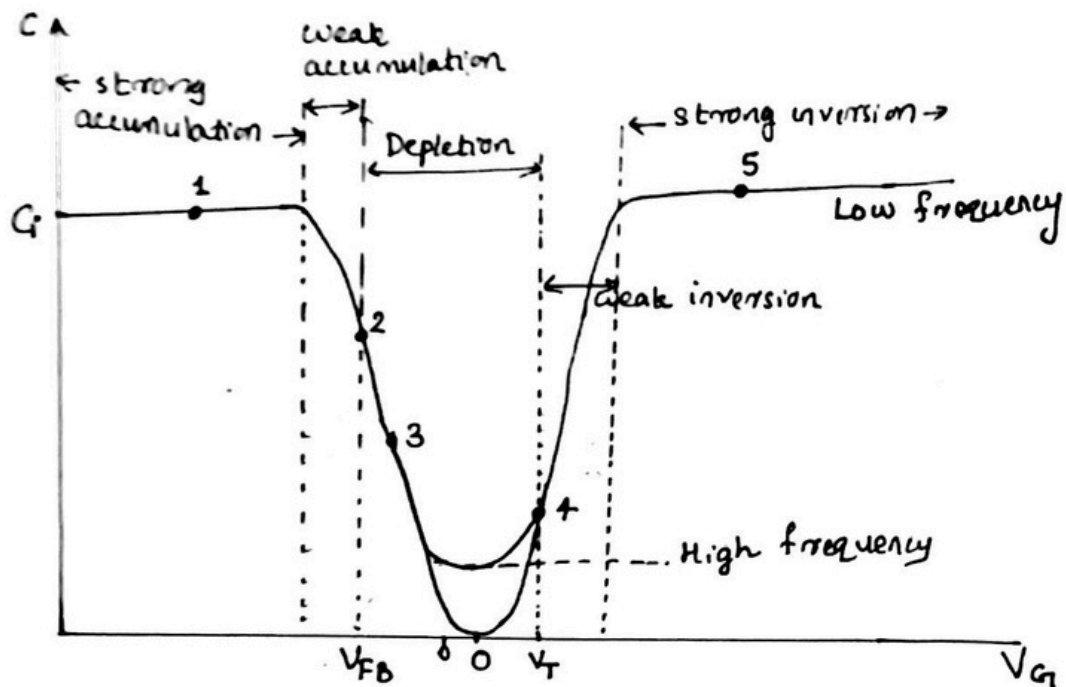
$$V_{Th} = V_{FB} + -\frac{Q_d}{C_{ox}} + 2\phi_F$$

$$V_{Th} = \underbrace{\phi_{ms} - \frac{Q_i}{C_{ox}}}_{\text{Flat band voltage}} - \underbrace{\frac{Q_d}{C_{ox}}}_{\text{Voltage for depletion layer formation}} + \underbrace{2\phi_F}_{\text{Voltage for inversion}} \rightarrow \text{In real MOS}$$

C-V characteristics

MOS device consist of 2 capacitances

- i) C_i - insulation capacitance or C_{ox} oxide capacitance between gate and channel.
- ii) C_d : depletion capacitance b/w channel and substrate



1. When $V_G < 0$

nmos in strong accumulation region i.e., more & more holes are accumulated near the surface. No depletion region is formed.

$C_d \rightarrow \text{negligible}$, $C_i \rightarrow \text{large}$

2. As V_G becomes less negative

C_d increases, C_i and C_d comes in series.

Total capacitance, $C = \frac{C_d \cdot C_i}{C_d + C_i}$, effective capacitance decrease

3. When $V_G = 0$ (equilibrium)

we obtain flat band voltages. Beyond this depletion region width increases.

$C_d \propto \frac{1}{\omega}$ so C_d decreases, thus C decreases.

~~increases~~ $V_G > 0$ until finally inversion is reached at V_T .

4. When $V_G > 0$, C_d decreases, threshold is attained.

5. After inversion C depends on the input frequency of the gate source voltage.

i) High frequency: If capacitance is measured with the high frequency signal, the inversion layer cannot respond to fast variation in the input signal amplitude. As a result effective capacitance remains constant at its minimum value.

ii) When capacitance is measured with low frequency signal, inversion layer charge varies with slow variation in the

input signal amplitude. The depletion layer widths remain unchanged. C_d is absent and C increases to C_i or C_{ox} .

Types of MOSFET

Depletion and Enhancement mode MOSFETS.

Enhancement mode MOSFET (E-MOSFET)

In this type, Substrate extends all the way to SiO_2 layer. There is no channel doped between S and D.

→ Channel is induced when V_{GS} is applied

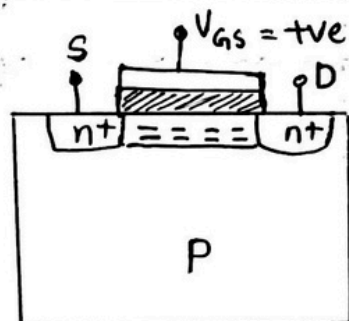
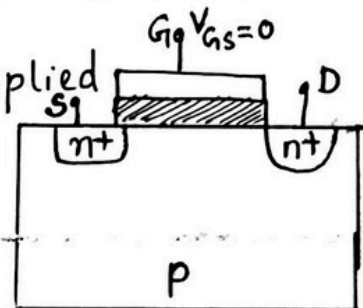
→ This MOSFET operates by inducing or enhancing channel.

→ Thus at $V_{GS} = 0\text{V}$, it does not conduct.

So it is called Normally OFF device.

→ I_D is produced when $V_{GS} > V_{th}$

→ As $V_{GS} > V_{th}$, inversion layer is formed.



Depletion Mode MOSFET (D-MOSFET)

→ In this type, there is a channel diffused between S and D at the time of fabrication.

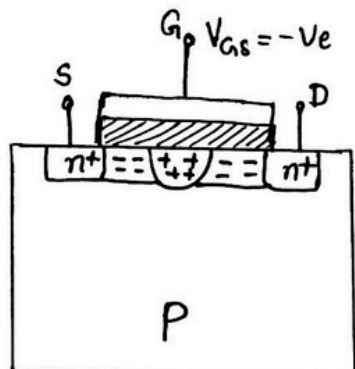
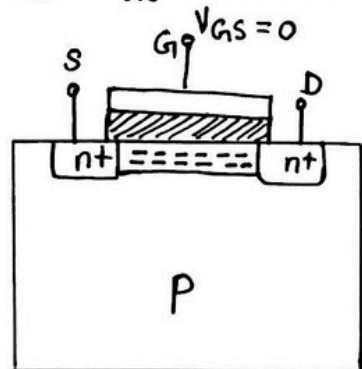
→ So at $V_{GS} = 0\text{V}$, Current can flow between S & D.

Thus it is called Normally ON device.

→ When $V_{GS} = -ve$, gate repels some of the -ve charge out of the N-channel. This increases the depletion region in this channel.

→ This increases R and decreases drain current.

→ As V_{GS} becomes more -ve, I_D decreases.

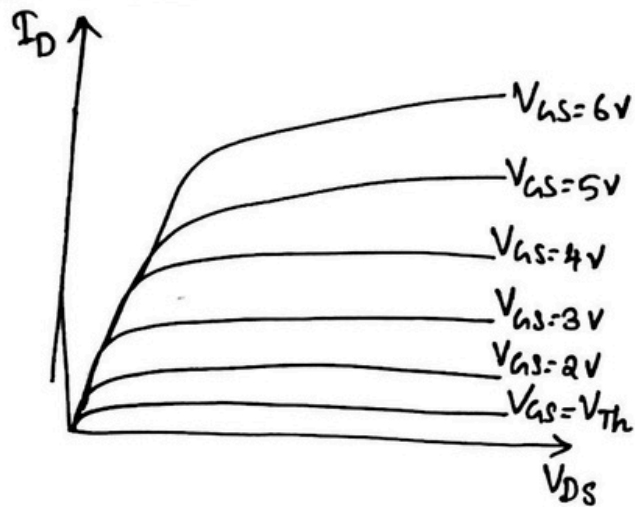


MOSFET characteristics:

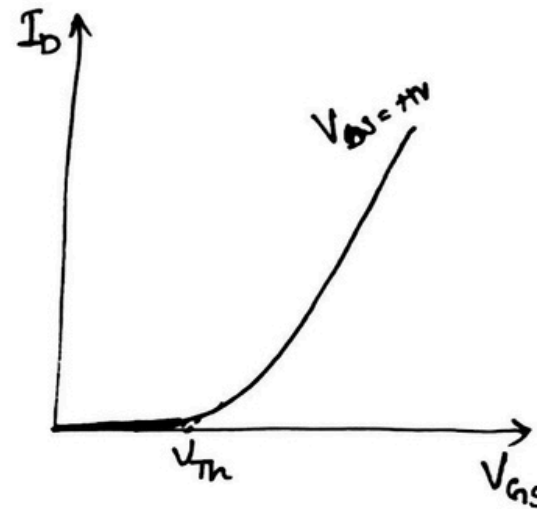
Output characteristics

nmos - Enhancement (CMOSFET)

1. Drain characteristics

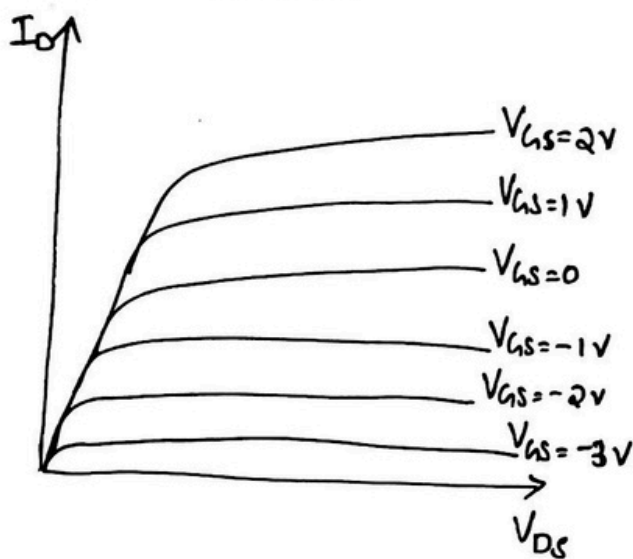


2. Transfer characteristics

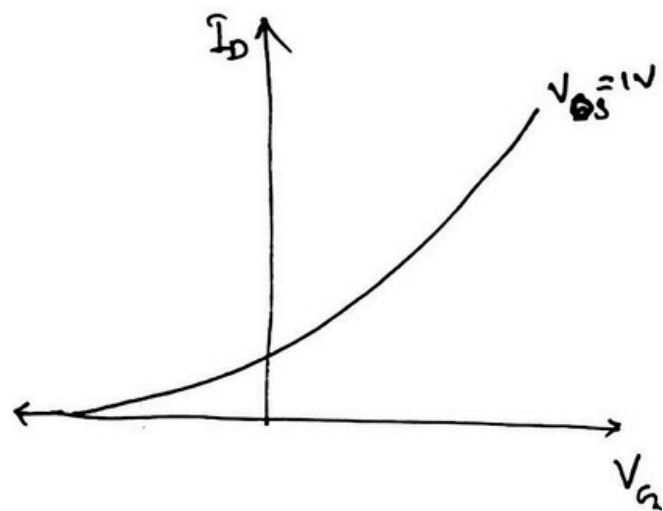


Depletion MOSFET - nmos (DMOSFET)

Drain characteristics



Transfer characteristics



Expression for drain current, I_D

we have,

$$\text{Gate voltage, } V_G = V_{FB} - \frac{Q_s}{C_{ox}} + \phi_s \quad \text{--- (1)}$$

$$\text{but } Q_s = Q_n + Q_d$$

Q_n : Charge to form inversion layer

Q_d : depletion layer charge

Neglect Q_d . So $Q_s = Q_n$.

$$Q_n = -C_{ox}(V_{GS} - V_T - V_{ox}) \quad \text{--- (2)}$$

The resistance of differential length dx of the channel at a distance ' x ' from the source is

$$R = \frac{\rho l}{A} \quad \text{--- (3)}$$

$$dR = \frac{-dx}{Q_n(x) \cdot \mu_n \cdot W} \quad \text{--- (4)}$$

Since the current through the channel is constant, the voltage drop dV_{ox} is

$$dV_{ox} = I_D \cdot dR = I_D \cdot \frac{-dx}{Q_n(x) \cdot \mu_n \cdot W} \quad \text{--- (5)}$$

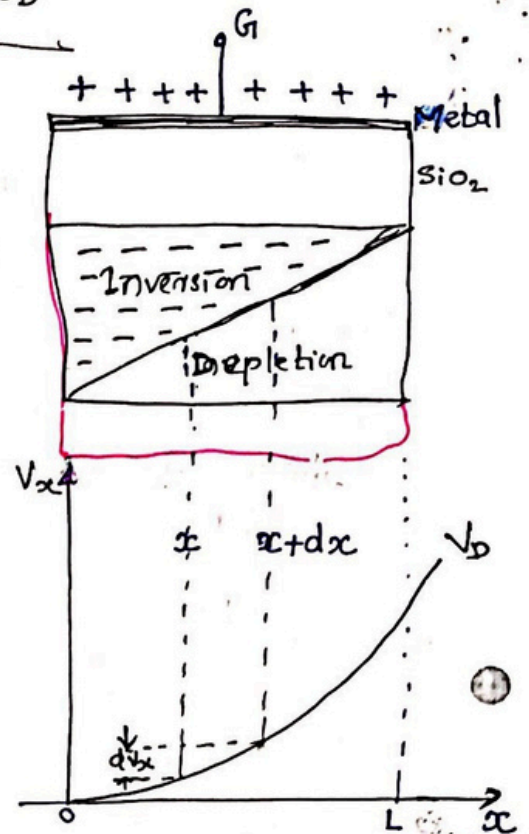
Substitute (2) in (5)

$$dV_{ox} = \frac{-I_D \cdot dx}{-C_{ox}(V_{GS} - V_T - V_{ox}) \cdot \mu_n \cdot W}$$

$$I_D \cdot dx = C_{ox} \cdot \mu_n \cdot W (V_{GS} - V_T - V_{ox}) \cdot dV_{ox} \quad \text{--- (6)}$$

Integrating (6)

$$\int_0^L I_D \cdot dx = C_{ox} \cdot \mu_n \cdot W \int_0^{V_{DS}} (V_{GS} - V_T - V_{ox}) \cdot dV_{ox}$$



$$\rho = \frac{1}{\sigma}$$

$$\sigma = -q \mu_n$$

$$A = wt, \quad t = 1$$

$$A = W \text{ or } Z$$

$$L = dx$$

$$R = \frac{\rho l}{A} = \frac{-dx}{q \mu_n W}$$

$$\therefore I_D \cdot L = C_{ox} \cdot \mu_n \cdot W \left[(V_{GS} - V_T) V_x - \frac{V_x^2}{2} \right]_0^{V_{DS}}$$

$$I_D \cdot L = C_{ox} \cdot \mu_n \cdot W \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$I_D = \frac{\mu_n \cdot C_{ox} \cdot W}{L} \cdot \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] \quad \text{--- (7)}$$

$$I_D = k_p \cdot \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] \quad \text{--- (8)}$$

where $k_p = \frac{\mu_n \cdot C_{ox} \cdot W}{L}$ --- (9)

The inversion layer charge decreases as the channel moves from source to drain. So the actual drain current will be less than that of the drain current obtained in (8)

Thus $V_{DS} = V_{D(sat)} = V_{GS} - V_T$ --- (10)

So as long as $V_{DS} \geq V_{DS(sat)}$

$$\begin{aligned} \text{(8)} \Rightarrow I_D &= k_p \cdot \left[(V_{GS} - V_T) (V_{GS} - V_T) - \frac{(V_{GS} - V_T)^2}{2} \right] \\ &= k_p \cdot \left[(V_{GS} - V_T)^2 - \frac{(V_{GS} - V_T)^2}{2} \right] \end{aligned}$$

$$I_{D(sat)} = \frac{k_p}{2} \cdot (V_{GS} - V_T)^2 \quad \text{--- (11)}$$

(i) If $V_{GS} > V_{Th}$ and $V_{DS} < V_{GS} - V_{Th}$: Linear region

$$I_D = \mu_n \cdot C_{ox} \cdot \frac{W}{L} \left[(V_{GS} - V_{Th}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

(ii) If $V_{GS} > V_{Th}$, $V_{DS} > V_{GS} - V_{Th}$;

$$I_D = \frac{\mu_n \cdot C_{ox} \cdot W}{2L} (V_{GS} - V_{Th})^2$$

Saturation region.

(iii) If $V_{GS} < V_{Th}$, $I_D = 0$.

Q: A silicon n channel MOSFET has $\mu_n = 600 \text{ cm}^2/\text{V}\cdot\text{s}$, $C_{ox} = 3 \times 10^{-7} \text{ F/cm}^2$, $W = 60 \mu\text{m}$, $L = 10 \mu\text{m}$, $V_T = 0.7 \text{ V}$. Find the drain current.

(i) When $V_{GS} = 3 \text{ V}$, $V_{DS} = 1 \text{ V}$

(ii) When $V_{GS} = 4 \text{ V}$, $V_{DS} = 6 \text{ V}$

Ans:
$$I_D = \frac{\mu_n C_{ox} \cdot W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$(i) V_{GS} - V_T = 3 - 0.7 = 2.3 \text{ V} > V_{DS}$$

$$I_D = \frac{\mu_n C_{ox} \cdot W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$= \frac{600 \times 3 \times 10^{-7} \times 60 \times 10^{-6}}{10 \times 10^{-6}} \left[(3 - 0.7) \times 1 - \frac{1}{2} \right]$$

$$= 0.00108 \text{ (1.8)}$$

$$= \underline{\underline{0.001944 \text{ A}}}$$

$$(ii) V_{GS} - V_T = 4 - 0.7 = 3.3$$

$$V_{GS} - V_T < V_{DS}$$

$$V_{DS} = 6 \text{ V}$$

$$I_D = \frac{\mu_n C_{ox} \cdot W}{2L} (V_{GS} - V_T)^2$$

$$= 0.00054 \text{ (10.89)}$$

$$= \underline{\underline{0.0058806 \text{ A}}}$$

2. A Si N channel MOSFET has $\mu_n = 600 \text{ cm}^2/\text{s}$,
 $C_{ox} = 1.2 \times 10^{-7} \text{ F/cm}^2$, $N = 50 \mu\text{m}$, $L = 10 \mu\text{m}$, $V_{th} = 0.8 \text{ V}$
 Find drain current.

'W' or 'Z' = width

1. When $V_{GS} = 2 \text{ V}$, $V_{DS} = 1 \text{ V}$.

2. When $V_{GS} = 3 \text{ V}$, $V_{DS} = 5 \text{ V}$

1) $V_{GS} = 2 \text{ V}$, $V_{DS} = 1 \text{ V}$

$$V_{GS} - V_T = 2 - 0.8 = 1.2 \gg V_{DS}$$

Linear region, $(V_{GS} - V_T \gg V_{DS})$

$$I_D = K_P \cdot \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$K_P = \frac{\mu_n \cdot C_{ox} \cdot W}{L} = \frac{600 \times 1.2 \times 10^{-7} \times 50 \times 10^{-6}}{10 \times 10^{-6}}$$

$$= 3.6 \times 10^{-4}$$

$$I_D = 3.6 \times 10^{-4} \left[(2 - 0.8) 1 - \frac{1^2}{2} \right] = 2.5 \times 10^{-4} \text{ A}$$

2) ~~$V_{GS} = 3 \text{ V}$~~ $V_{GS} = 3 \text{ V}$, $V_{DS} = 5 \text{ V}$

~~$$V_{GS} - V_T = 3 - 0.8 = 2.2 < V_{DS}$$~~

Saturation region, $(V_{GS} - V_T < V_{DS})$

$$I_D = \frac{K_P}{2} (V_{GS} - V_T)^2$$

$$= \frac{3.6 \times 10^{-4}}{2} [3 - 0.8]^2 = 8.7 \times 10^{-4} \text{ A}$$

1. A Si MOS structure at 300K is characterised by

$N_A = 4 \times 10^{14} \text{ cm}^{-3}$. The SiO_2 thickness is $t_{\text{ox}} = 0.1 \mu\text{m}$.

Oxide is ideal. Find the width of the depletion layer and threshold voltage. E_r of Si and SiO_2 are 11.8 and 3.9 respectively.

$$V_T = -\frac{Q_d}{C_{\text{ox}}} + \phi_{s(\text{inv})} = -\frac{Q_d}{C_{\text{ox}}} + 2\phi_F$$

$$\begin{aligned} \phi_F &= \frac{kT}{q} \ln \frac{N_A}{n_i} \\ &= \frac{8.26 \times 10^{-5} \times 300}{1} \cdot \ln \left(\frac{4 \times 10^{14}}{1.5 \times 10^{10}} \right) = \underline{\underline{0.26}} \end{aligned}$$

$$\begin{aligned} W_m &= 2 \sqrt{\frac{\epsilon \phi_F}{2N_A}} = 2 \sqrt{\frac{\epsilon_0 \epsilon_r \phi_F}{2N_A}} \\ &= 2 \sqrt{\frac{8.854 \times 10^{-14} \times 11.8 \times 0.26}{1.6 \times 10^{-19} \times 4 \times 10^{14}}} = \underline{\underline{1.3 \times 10^{-4} \text{ cm}}} \end{aligned}$$

$$Q_d = -2N_A \cdot W_m = -1.6 \times 10^{-19} \times 4 \times 10^{14} \times 1.3 \times 10^{-4} = -8.43 \times 10^{-9} \text{ C}$$

$$C_{\text{ox}} = \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}} = \frac{\epsilon_0 \epsilon_{r(\text{oxide})}}{t_{\text{ox}}} = \frac{8.854 \times 10^{-14} \times 3.9}{0.1 \times 10^{-4}} = 3.45 \times 10^{-8} \text{ F/cm}^2$$

$$\begin{aligned} V_T &= \frac{8.43 \times 10^{-9}}{3.45 \times 10^{-8}} + 2 \times 0.26 \\ &= \underline{\underline{0.773 \text{ V}}} \end{aligned}$$

Find the maximum width of the depletion region for an ideal MOS capacitor on p-type Si with $N_a = 10^{16} \text{ cm}^{-3}$.

EXAMPLE 6-1

The relative dielectric constant of Si is 11.8 from Appendix III. We get ϕ_F from Eq. (6-15):

SOLUTION

$$\phi_F = \frac{kT}{q} \ln \frac{N_a}{n_i} = 0.0259 \ln \frac{10^{16}}{1.5 \times 10^{10}} = 0.347 \text{ V} \quad \checkmark$$

Thus

$$\begin{aligned} W_m &= 2 \sqrt{\frac{\epsilon_s \phi_F}{q N_a}} = 2 \left[\frac{(11.8)(8.85 \times 10^{-14})(0.347)}{(1.6 \times 10^{-19})(10^{16})} \right]^{1/2} \\ &= 3.01 \times 10^{-5} \text{ cm} = 0.301 \text{ } \mu\text{m} \end{aligned}$$

EXAMPLE 6-2

Using the conditions of Example 6-1 and a 100-Å-thick SiO₂ layer, we can calculate major points on the C - V curve of Fig. 6-16. The relative dielectric constant of SiO₂ is 3.9.

$$C_i = \frac{\epsilon_i}{d} = \frac{(3.9)(8.85 \times 10^{-14})}{10^{-6}} = 3.45 \times 10^{-7} \text{ F/cm}^2$$

$$Q_d = -qN_aW_m = -(1.6 \times 10^{-19})(10^{16})(0.301 \times 10^{-4}) \\ = -4.82 \times 10^{-8} \text{ C/cm}^2$$

$$V_T = -\frac{Q_d}{C_i} + 2\phi_F = \frac{4.82 \times 10^{-8}}{34.5 \times 10^{-8}} + 2(0.347) = 0.834 \text{ V}$$

At maximum depletion

$$C_d = \frac{\epsilon_s}{W_m} = \frac{(11.8)(8.85 \times 10^{-14})}{0.301 \times 10^{-4}} = 3.47 \times 10^{-8} \text{ F/cm}^2$$

$$C_{\min} = \frac{C_i C_d}{C_i + C_d} = \frac{34.5 \times 3.47}{34.5 + 3.47} 10^{-8} = 3.15 \times 10^{-8} \text{ F/cm}^2$$

We can calculate V_T for the MOS structure described in Examples 6-1 and 6-2, including the effects of flat band voltage. If n^+ polysilicon is used for the gate, Fig. 6-17 indicates $\Phi_{ms} = -0.95$ V for $N_a = 10^{16} \text{ cm}^{-3}$. Assuming an interface charge of $5 \times 10^{10} \text{ q}(\text{C}/\text{cm}^2)$, we obtain

EXAMPLE 6-3

$$\begin{aligned} V_T &= \Phi_{ms} + 2\phi_F - \frac{1}{C_i}(Q_i + Q_d) \\ &= -0.95 + 0.694 - \frac{(5 \times 10^{10} \times 1.6 \times 10^{-19}) - 4.82 \times 10^{-8}}{34.5 \times 10^{-8}} \\ &= -0.14 \text{ V} \end{aligned}$$

This value corresponds to the $N_a = 10^{16} \text{ cm}^{-3}$ point in Fig. 6-20 for the n-channel case.

ECT 201 SSD

MODULE 5

MOSFET scaling – need for scaling, constant voltage scaling and constant field scaling. Sub threshold conduction in MOS. Short channel effects- Channel length modulation, Drain Induced Barrier Lowering, Velocity Saturation, Threshold Voltage Variations and Hot Carrier Effects.
Non-Planar MOSFETs: Fin FET –Structure, operation and advantages.

MOSFET SCALING

Scaling of a MOS transistor means reducing the critical parameters of the device in accordance with a given criterion in order to improve some performance features such as Speed, Application, Power Dissipation, and so on while keeping the basic operational characteristics unchanged.

scaling requires all device dimensions to reduce proportionally. The main device dimensions are the channel length, channel width, and oxide thickness.

Lateral dimensions such as channel length and width are reduced by a factor of k , so should the vertical dimensions such as source/drain junction depths and gate insulator thickness

Scaling of depletion width is achieved indirectly by scaling up doping concentrations. If we simply reduce the dimensions of the device and kept the power supply voltages same, the internal electric field in the device would increase. **Need for scaling Scaling improves**

1. Packing density: The packing density of the device improves as a result of scaling hence we can fit more transistors in the same space as before.

2. Speed

3. Power dissipation

Two types of scaling are common:

(i) constant field scaling

(ii) constant voltage scaling.

Full scaling (constant-field scaling) –

- All dimensions are scaled by k and the supply voltage and other voltages are so scaled
- Magnitude of internal electric field is kept constant

- Only lateral dimensions are changed
- Threshold voltage is also affected

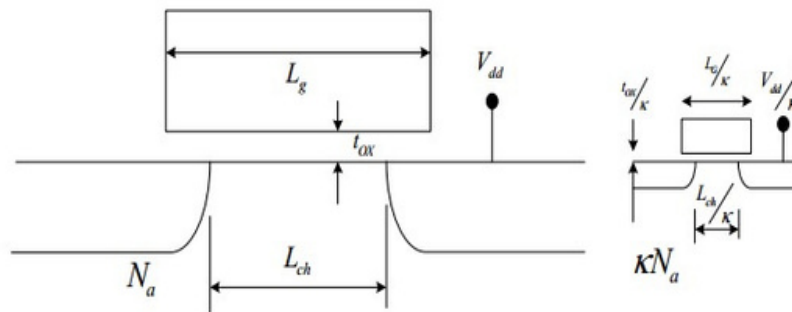


Figure 2: 5 Illustration of MOSFET miniaturisation. The sketch on the right hand is the scaled device according to the constant field rule. (Reference [2.15])

Constant field scaling yields the largest reduction in the power-delay product of a single transistor. However, it requires a reduction in the power supply voltage as one decreases the minimum feature size.

For ideal scaling, power supply voltages should be reduced to keep the internal electric field reasonably constant from one technology generation to the next. But power supply voltages are not scaled hand in hand with the device dimensions, partly because of other system related constraints. The longitudinal electric field in the pinch off region and the transverse electric field across the gate oxide increase with MOSFET scaling which causes **hot electron effects and short channel effects**.

Table 6-1 Scaling rules for MOSFETs according to a constant factor K . The horizontal and vertical dimensions are scaled by the same factor. The voltages are also scaled to keep the internal electric fields more or less constant, and the hot carrier effects manageable.

	Scaling factor
Surface dimensions (L, Z)	$1/K$
Vertical dimensions (d, x_j)	$1/K$
Impurity Concentrations	K
Current, Voltages	$1/K$
Current Density	K
Capacitance (per unit area)	K
Transconductance	1
Circuit Delay Time	$1/K$
Power Dissipation	$1/K^2$
Power Density	1
Power-Delay Product	$1/K^3$

Constant voltage scaling does not have this problem and is therefore the preferred scaling method since it provides voltage compatibility with older circuit technologies.

Constant-voltage scaling: In Constant voltage scaling, all dimensions of the MOSFET are reduced by a factor of 'k', but power supply & terminal voltage remain unchanged.

The voltages are not scaled and, in some cases, dimensions associated with voltage are not scaled.

The disadvantage of constant voltage scaling is that the electric field increases as the minimum feature length is reduced.

Constant voltage scaling

Parameter	Scaled parameter
Channel length (L)	$1/\alpha$
Junction depth (x_j)	$1/\alpha$
Substrate doping (N_A)	α
Depletion layer thickness (d)	$1/\alpha$
Transconductance (g_m)	α
Static power dissipation (P_{stat})	α
Dynamic power dissipation (P_{dyn})	α
Current (I)	α
Gate delay (τ_p)	$1/\alpha^2$
Load capacitance (C_L)	$1/\alpha$
Channel width (W)	$1/\alpha$
Supply voltage (V)	1
Gate oxide thickness (t_{ox})	$1/\alpha$
Current density (J)	α^3

Comparison

Quantity	Sensitivity	Constant Field	Constant Voltage
Scaling Parameters			
Length	L	$1/S$	$1/S$
Width	W	$1/S$	$1/S$
Gate Oxide Thickness	t_{ox}	$1/S$	$1/S$
Supply Voltage	V_{dd}	$1/S$	1
Threshold Voltage	V_{T0}	$1/S$	1
Doping Density	NA, N^D	S	S^2
Device Characteristics			
Area (A)	WL	$1/S^2$	$1/S^2$
D-S Current (I_{DS})	$\propto (V_{ds} - V_{T0})^2$	$1/S$	S
Gate Capacitance (C_g)	WL/t_{ox}	$1/S$	$1/S$
Power Dissipation (P)	$I_{DS}V_{dd}$	$1/S^2$	S
Power Dissipation Density (P/A)	P/A	1	S^3

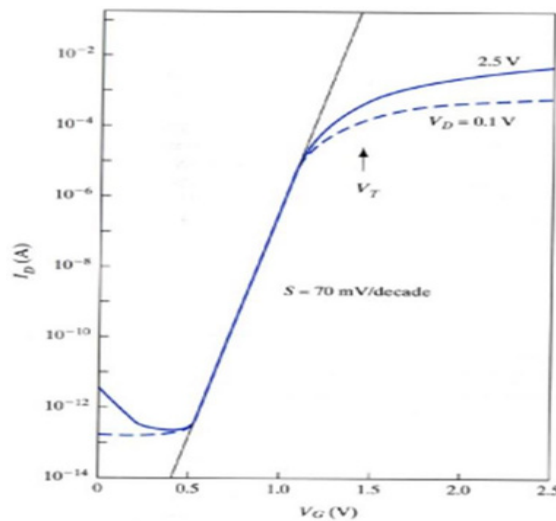
Subthreshold conduction / Subthreshold Characteristics

For the conduction to happen in the MOSFET; we need 'VG' to be greater than the threshold voltage VT. But this threshold voltage is calculated at the point where the region below the oxides has entered into strong inversion.

From experimental results, one can observe that there is still some non-zero current flowing from drain to source even when we are operating at a region with $V_G < V_T$ (sub-threshold region). This happens because, for the subthreshold region, the substrate near oxide-interface is in "Weak-Inversion". At this point, if we apply a positive VDS, there will be a small current I_D flowing. This effect is plotted in the transfer characteristics in figure below. We have,

$$I_D = \frac{W}{L} \cdot \mu_n C_{OX} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

In this equation, current abruptly goes to zero as soon as VG is reduced to VT. In reality there is still some drain conduction below threshold, and is known as subthreshold conduction. This current is due to weak inversion in the channel between flat band and threshold which leads to a diffusion current from source to drain.



Subthreshold Swing,

$$S = \log \frac{d(V_G)}{d(\log I_D)}$$

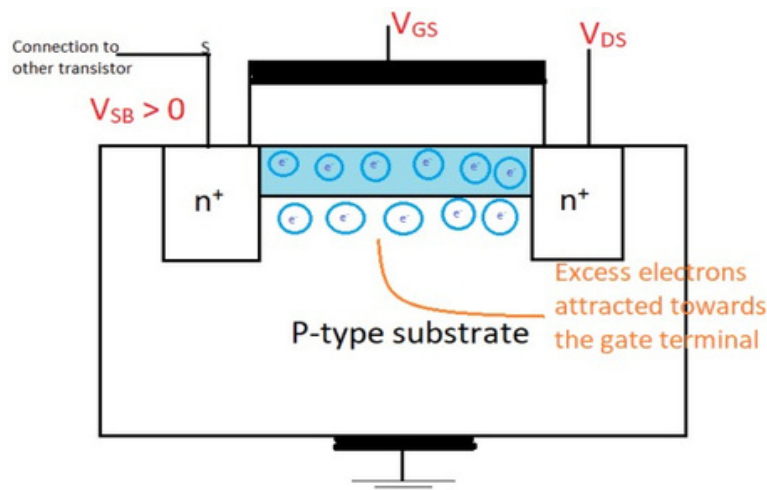
- Generally, in order to improve the performance and reduce the cost of production, one would prefer to scale down the size of the transistors.
- This scaling down also eliminates many stray capacitances that are present in the overall device. Ultimately increasing the speed of operation.
- But when the channel length is scaled down to the order of the depletion layer, a certain number of non-ideal effects come into play. These are called **second-order effects**

Substrate Bias Effect

For the ideal I-V characteristics, the biasing scheme we used had the source and the body both connected to ground.

But in practical design applications, Source is connected to substrate(body) so that there is a voltage V_{SB}

In such scenarios, the difference in potential between the body and the source terminal causes a change in the threshold voltage of the MOSFET. This effect of change in threshold voltage is called the “Body Effect” or the “Back Gate Effect”.



When V_{SB} is positive, there is reverse bias between source and bulk. This causes depletion layer to widen.

The electrons in the bulk are repelled by the body terminal and are now attracted by the gate toward the oxide layer.

The threshold voltage of the MOS is also proportional to the density of electrons in the depletion layer.

Hence as we accumulate more and more electrons in the depletion layer below the oxide interface, there will be an increase in the value of threshold voltage.

As depletion region is widened, larger charge density is occupied. Therefore, the threshold required to achieve inversion increases.

$$V_{TN} = V_{T0} + \gamma(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|})$$

V_{T0} = zero - substrate - bias V_T
 γ = body effect parameter
 $|\phi_F|$ = surface potential parameter

Short Channel Effects

Short-channel effects occur in MOSFETs in which the channel length is comparable to the depletion layer widths of the source and drain junctions.

A MOSFET device is considered to be short channel device when the channel length is the same order of magnitude as the depletion-layer widths (x_{dD} , x_{dS}) of the source and drain junction. (That is, the effective channel length L_{eff} is approximately equal to the source and drain junction depth x).

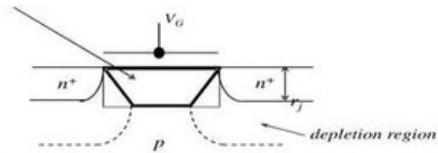
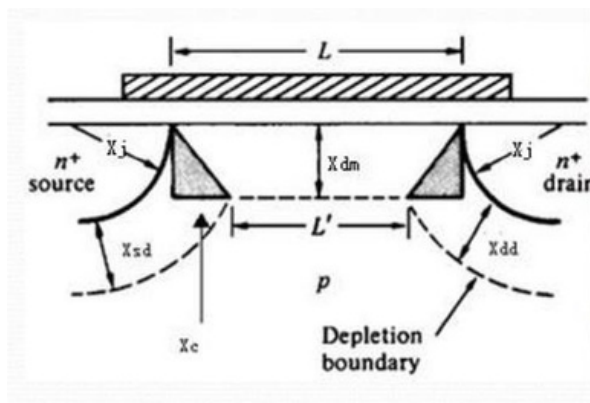
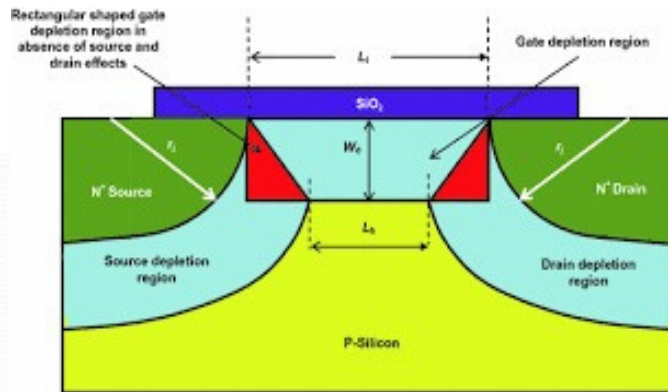
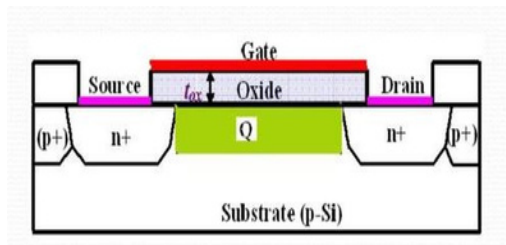
As the channel length L is reduced to increase both the operation speed and the number of components per chip, the so-called short-channel effects arise.

The short-channel effects are attributed to two physical phenomena:

1. The limitation imposed on electron drift characteristics in the channel
2. The modification of the threshold voltage due to the shortening channel length.

This occurs due to the charge sharing between source/drain and gate. A triangle region forms at both ends

Hence the rectangular area under the gate becomes Trapezoid



Different short-channel effects include

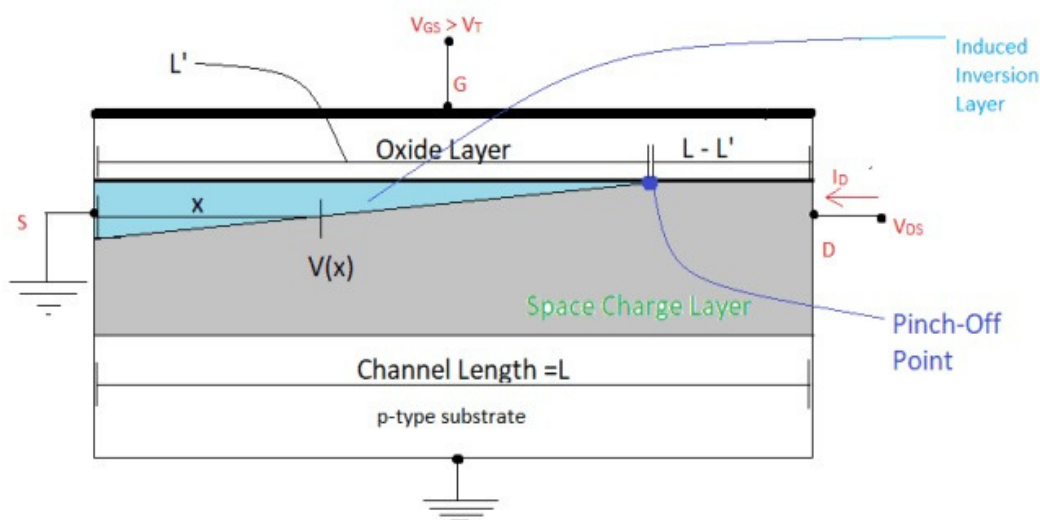
1. Channel Length Modulation
2. Drain-induced barrier lowering and “Punch through”
3. Velocity saturation
4. Threshold voltage variations
5. Hot carrier effects

Channel Length Modulation(CLM)

As we keep on increasing V_{DS} , the region for which the inversion charge is zero keeps on increasing for a constant value of V_{GS} maintained. Thus channel length keeps on decreasing. This phenomenon is called Channel Length Modulation.

This is similar to “Base Width Modulation” Thus we get a V_{DS} term in the expression for I_D even when we are operating in the saturation region.

Generally, the fabrication of the MOSFET devices is done in a way such that the change in length given by $\Delta L = L - L'$ is low with a change in V_{DS} .



Drain Induced Barrier Lowering (DIBL)

When the depletion regions surrounding the drain extends to the source, so that the two-depletion layer merge (i.e., when $x_{dS} + x_{dD} = L$), punch through occurs.

Punch through can be minimized with thinner oxides, larger substrate doping, shallower junctions, and obviously with longer channels.

The current flow in the channel depends on creating and sustaining an inversion layer on the surface.

If the gate bias voltage is not sufficient to invert the surface ($V_{GS} < V_T$), the carriers (electrons) in the channel face a potential barrier that blocks the flow. Increasing the gate voltage reduces this

potential barrier and, eventually, allows the flow of carriers under the influence of the channel electric field.

In small-geometry MOSFETs, the potential barrier is controlled by both the gate-to-source voltage V_{GS} and the drain-to-source voltage V_{DS} .

If the drain voltage is increased, the potential barrier in the channel decreases, leading to drain-induced barrier lowering (DIBL).

The reduction of the potential barrier eventually allows electron flow between the source and the drain, even if the gate-to-source voltage is lower than the threshold voltage.

The channel current that flows under this condition ($V_{GS} < V_T$) is called the sub-threshold current

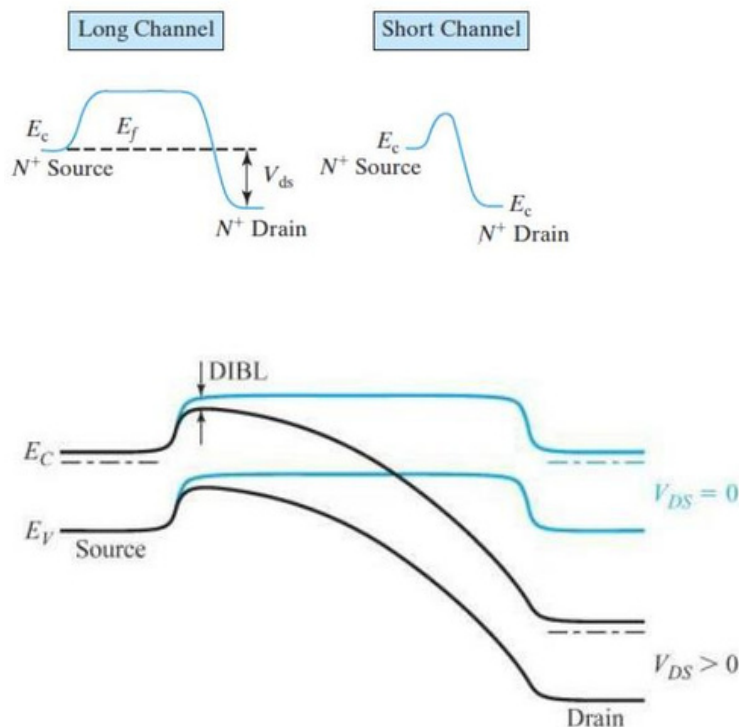


Figure 8.15 Energy-band diagrams along the channel of a MOSFET illustrating the drain-induced barrier lowering (DIBL).

Velocity Saturation

The velocity of charge carriers, such as electrons or holes, is proportional to the electric field that drives them, but that is only valid for small fields.

As the field gets stronger, their velocity tends to saturate. That means that above a critical electric field, they tend to stabilize their speed and eventually cannot move faster.

Velocity saturation is specially seen in short-channel MOSFET transistors, because they have higher electric fields

The drift velocity of the electrons in the inversion layer to be proportional to the lateral electric field applied. The proportionality constant was given by μ_n .

The key point to understand the effect of velocity saturation is that the linearity of the drift velocity only holds true for low values of the applied electric field. The actual variation of drift velocity with respect to the applied electric field is shown in figure 6.

The exact formula for the drift velocity can be given as:

$$v_d = \frac{\mu E}{1 + E/E_c}$$

The term E_c called the critical electric field. Here the electric field E is equal to $\frac{V_{DS}}{L}$, the lateral voltage applied across the channel divided by the effective channel length. We can see that for large channel devices, the drift velocity formula simplifies to $v_d = \mu E$. Hence this is also a short channel effect because the lateral electric field is higher in case of short channel devices for similar range of drain-to-source voltage applied.

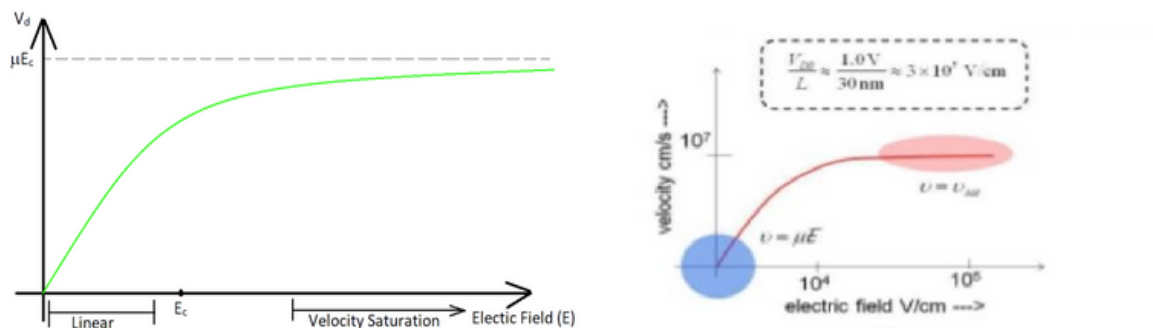


Figure 6: Variation of drift velocity of electron w.r.t. applied electric field

Threshold Variations

The threshold voltage is only a function of the manufacturing technology and the applied body bias VSB.

The threshold can therefore be considered as a constant over all NMOS (PMOS) transistors in a design. As the device dimensions are reduced, this model becomes inaccurate, and the threshold potential becomes a function of L , W , and V_{DS} .

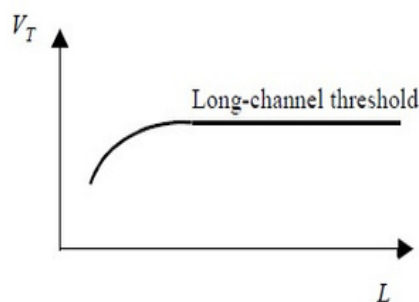
Two-dimensional second-order effects that were ignorable for long-channel devices suddenly become significant. In the traditional derivation of the VTO, for instance, it is assumed that the channel depletion region

is solely due to the applied gate voltage and that all depletion charge beneath the gate originates from the MOS field effects.

This ignores the depletion regions of the source and reverse-biased drain junction, which become relatively more important with shrinking channel lengths.

Since a part of the region below the gate is already depleted (by the source and drain fields), a smaller threshold voltage suffices to cause strong inversion.

In other words, V_{T0} decreases with L for short-channel devices (Figure 3.35a).



(a) Threshold as a function of the length (for low V_{DS})

Figure 3.35 Threshold variations.

Hot-Carrier Effects

Another problem, related to high electric fields, is caused by so-called hot electrons. This high energy electrons can enter the oxide, where they can be trapped, giving rise to oxide charging that can accumulate with time and degrade the device performance by increasing V_T and affect adversely the gate's control on the drain current.

Besides varying over a design, threshold voltages in short-channel devices also have the tendency to drift over time. This is the result of the hot-carrier effect

Over the last decades, device dimensions have been scaled down continuously, while the power supply and the operating voltages were kept constant. The resulting increase in the electrical field strength causes an increasing velocity of the electrons, which can leave the silicon and tunnel into the gate oxide upon reaching a high-enough energy level.

Electrons trapped in the oxide change the threshold voltage, typically increasing the thresholds of NMOS devices, while decreasing the V_T of PMOS transistors.

For an electron to become hot, an electrical field of at least 104 V/cm is necessary. This condition is easily met in devices with channel lengths around or below 1 μm .

The hot-electron phenomenon can lead to a long-term reliability problem, where a circuit might degrade or fail after being in use for a while. This is illustrated in Figure 3.36, which shows the degradation in the I-V characteristics of an NMOS transistor after it has been subjected to extensive operation.

MOSFET technologies, therefore use specially-engineered drain and source regions to ensure that the peaks in the electrical fields are bounded, hence preventing carriers to reach the critical values necessary to become hot.

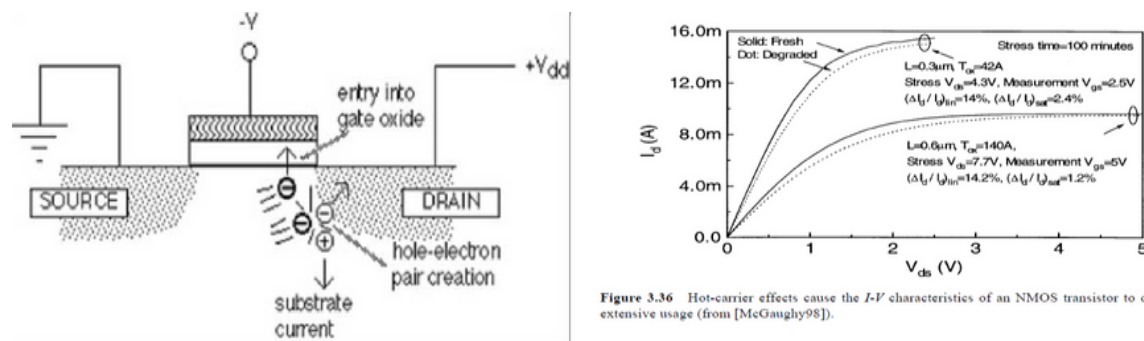


Figure 3.36 Hot-carrier effects cause the I-V characteristics of an NMOS transistor to degrade from extensive usage (from [McGaughy98]).

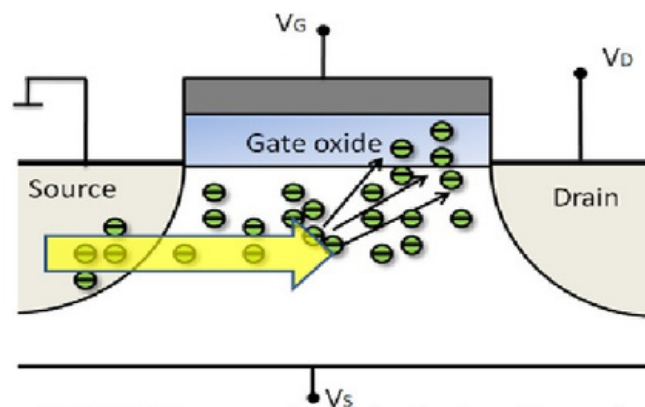


Figure 1. MOSFET cross-sectional visualization of hot carrier effect.

FinFET

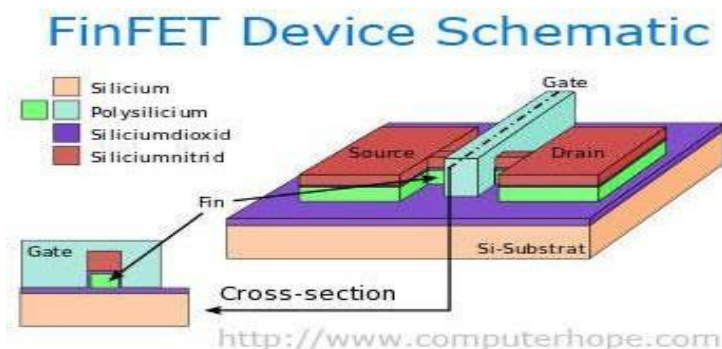
- FinFET, also known as Fin Field Effect Transistor, is a type of **non-planar** or **"3D" transistor** used in the design of modern processors
- FinFETs are new generation transistors which utilize **tri-gate structure**. In contrast to planar transistors where the Gate electrode was (usually) above the channel, the Gate electrode **"wraps" the channel**. The distinguishing characteristic of the FinFET is that the conducting channel is wrapped
- by a thin silicon "fin", which forms the body of the device.
- The conducting channel is greatly controlled by the gate.
- The thickness of the fin (measured in the direction from source to drain) determines the effective channel length of the device.
- These effects make it harder for the voltage on a gate electrode to deplete the channel underneath and stop the flow of carriers through the channel – in other words, to turn the transistor Off. By raising the channel above the surface of the wafer instead of creating the channel just below the surface, it is possible to wrap the gate around up to three of its sides, providing much greater electrostatic control over the carriers within it.

Advantages

Reduced short-channel effects (SCEs) and **leakage current** .

To overcome the worst types of short-channel effect encountered by deep submicron transistors, such as drain-induced barrier lowering (DIBL).

This technique provides increased operating speed by low-threshold MOSFET and **reduced leakage** by high-threshold voltage.



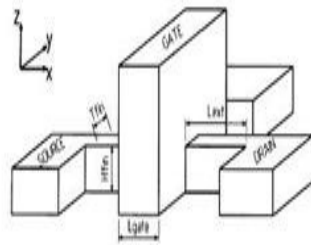
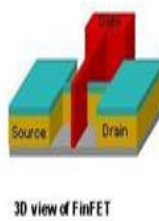
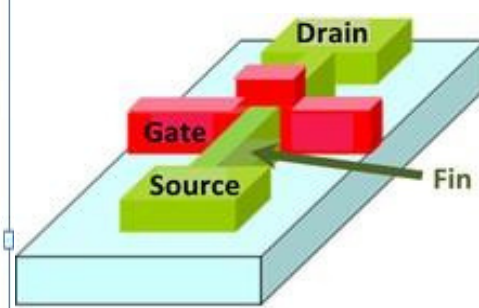


Fig. 1. Schematic of a FinFET structure



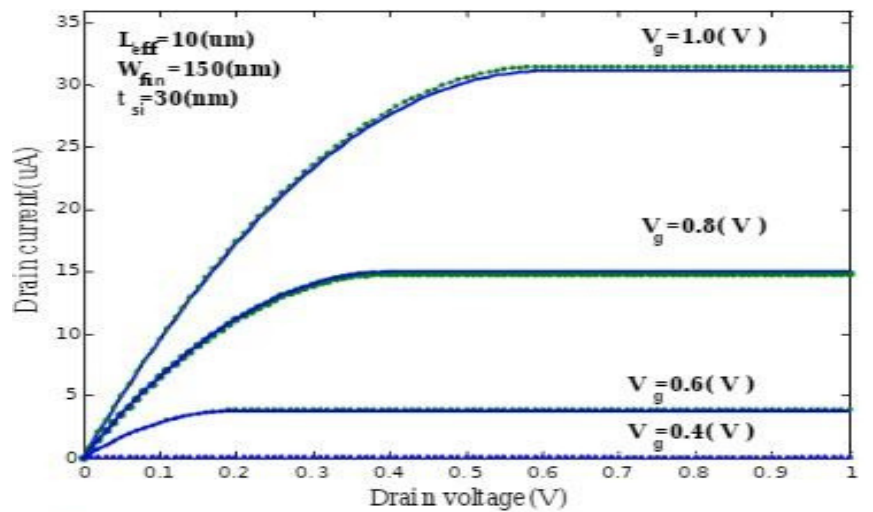
3D view of FinFET



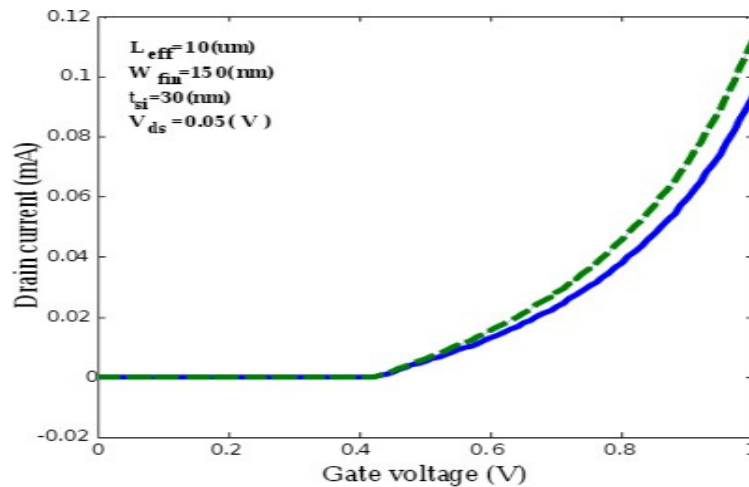
- The very first finFETs were manufactured on top of insulating layer.
- The fact that the current can't flow "underneath" the gate when the transistor is in OFF state reduces the leakage current.
- Alternative techniques for stopping leakage current from flowing in the bulk were introduced later, which allowed for manufacturing of Bulk finFETs. This technique utilizes very high doping gradients along the height of the fin in order to prevent the current from flowing in the bulk.

Output Characteristics

Drain Characteristics



Transfer Characteristics



PREVIOUSYEAR QUESTIONS

DECEMBER 2020

1. What is channel length modulation in MOSFETs? How does it affect the output characteristics of the MOSFET? (3)
2. Explain the principle of operation and advantage of FinFET.
3. What is meant by scaling in MOSFETs? Explain the challenges in device scaling? (7)
4. Explain the concept of constant voltage scaling and its limitations. (7)
5. What is meant by DIBL in MOSFETs? How does it affect the threshold voltage of a MOSFET? (7)
6. Explain the concepts of velocity saturation and hot carrier effects in a MOSFET. (7)

DECEMBER 2021

1. Explain Drain induced barrier lowering? (3)
2. Draw and label the structure of a FINFET (3)
3. Distinguish between constant voltage scaling and constant field scaling (8)
4. Illustrate the operation of FinFET (6)
5. Explain any four short channel effects in MOSFET (14)
